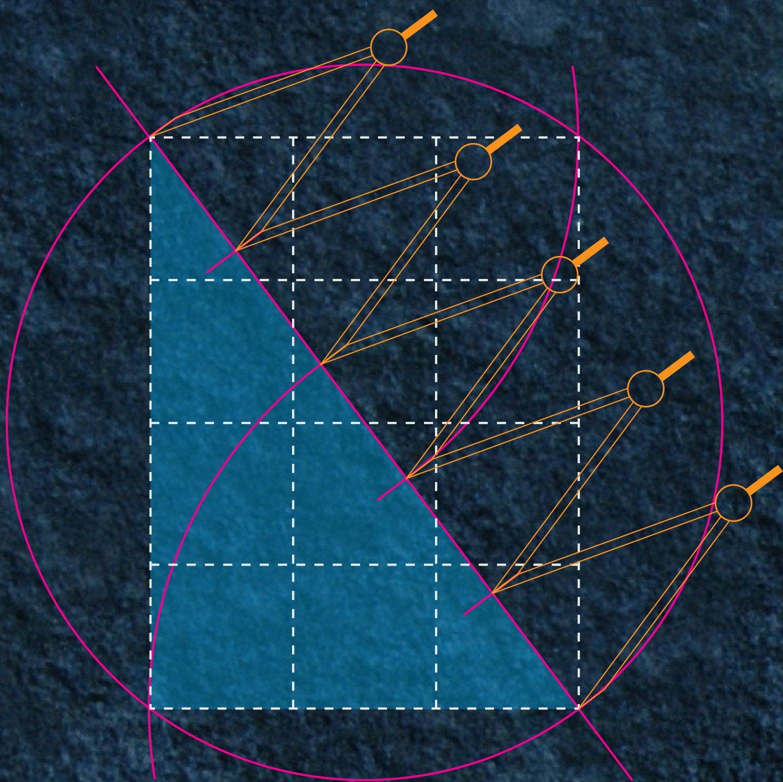
22 Useful Geometries for Carpenters



Laurie SMITH

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HISTORIC BUILDING GEOMETRY
devoted to geometrical learning

The three traditional tools used for hand drawn geometrical construction were dividers, straight edge and scribe, historic precursors of the modern compass, ruler and pen (*or pencil*). The only difference between the straight edge and ruler is that the straight edge, which functions solely as a guide for drawing straight lines, does not have measurements marked on its surface. The geometrical constructions in this pamphlet are described as if drawn manually using a compass, straight edge and pen on pure white paper. The reader is encouraged to draw the constructions because drawing is the best route to understanding.

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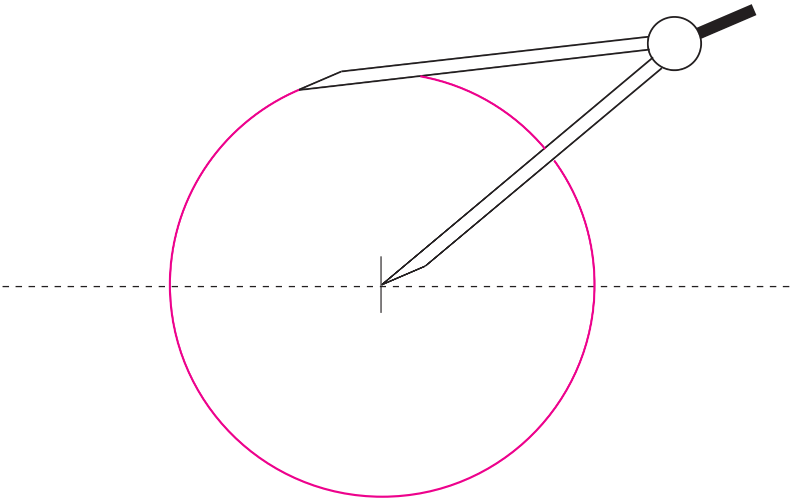
HISTORIC BUILDING GEOMETRY

22
useful geometries for carpenters

The geometries start with the simplest and are shown first as step by step constructions and then, if applicable, as short cut methods that give the same geometrical results. Some of the geometries commence from compass construction and can be categorised as circle geometry while others, developmented from the square, can be thought of as square geometry. However, the simplest, fastest and most accurate way to construct a square is by compass geometry and straight edge and this reveals a fundamental fact of geometrical design: that circle and square geometries are often interdependent. It should also be recognised that, just as constructing a building requires scaffolding, certain geometrical developments function as scaffolding and are removed after serving their purpose.

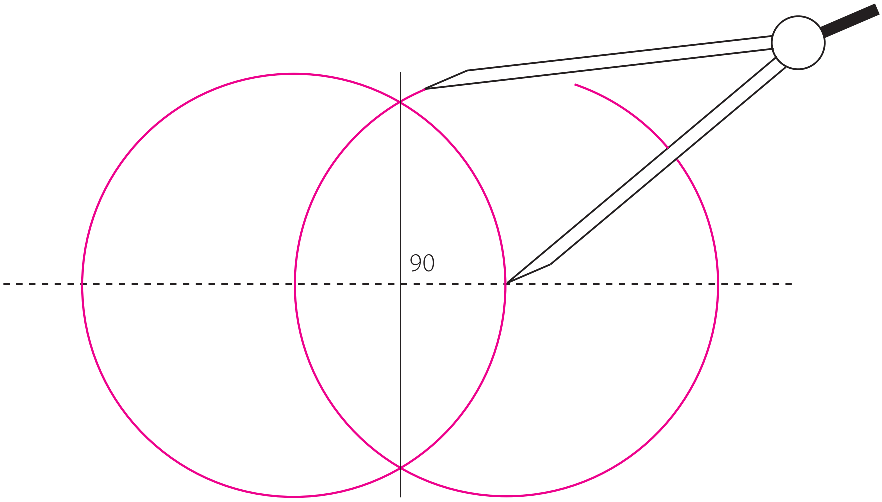
Point, line and plane

Geometry begins with the point, line and plane. A point is best described as a location without dimension, the best practical example being the pinprick of a compass point into the surface of a sheet of paper. Line can be thought of as the connection between two points and, though it has length, like the point it has no dimension. The edge of a sheet of paper, for example, is a line without dimension, a division between the tangible paper and the space around it. A plane exists within linear two-dimensional boundaries such as a circle (single curved line), a vesica piscis (two curved lines), triangle (three straight lines), square, rectangle or parallelogram (four straight lines), etc. It is clear from these examples that a line can be either straight or curved so that a circle passing through all four corners of a square is simultaneously a square that meets a circle at four points, an example of two different geometrical routes between the four equidistant points. A circle's continuous single line curvature has a straight line as its diameter (or more specifically as its rotating radius) and a point at its axis so that the circle is the simplest and most perfect embodiment of point, line and plane.



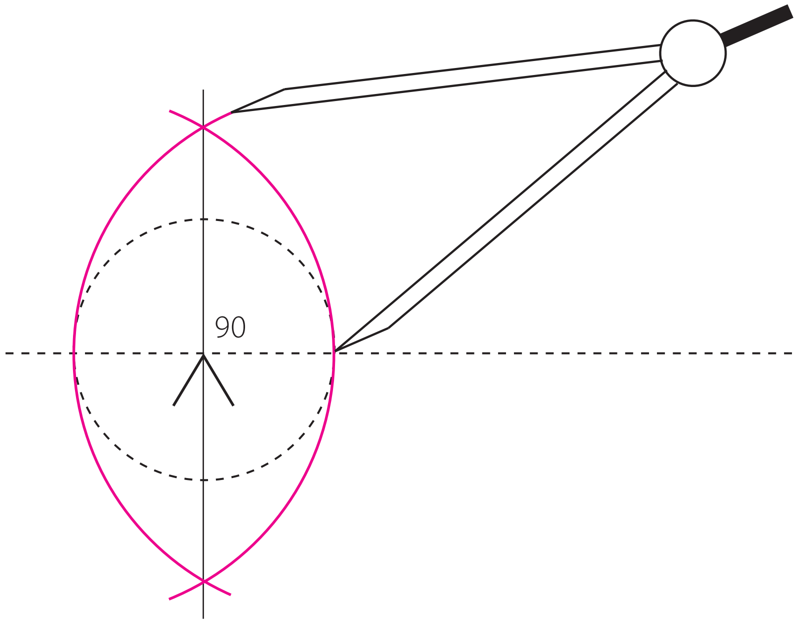
1 Drawing a circle

Anyone can draw a circle. However, the most useful circle is best drawn from a point on a straight line. This is because the resulting circle automatically has a diameter and, in consequence, three precision points, the circle's axis and the two points on the circumference at opposite ends of the diameter. The three points are useful in constructions that require more than one circle. Also, the diameter passes exactly through the circle's centre and is, therefore, a centre line for any further geometrical construction.



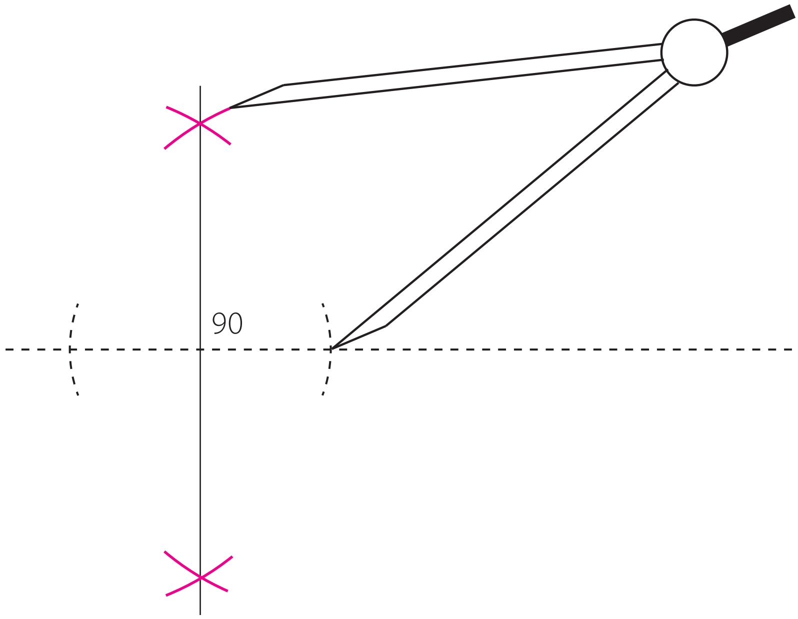
2 Drawing a perpendicular

A perpendicular is a line drawn precisely at 90° to another so, if a circle is drawn from a point on a line, then one of the two lines is already in place. If a second circle (of identical radius to the first) is drawn from where the diameter / centre line cuts the first circle, the two circles form a vesica piscis (the mandorla at the centre of the two circles). A vertical line drawn through the poles of the vesica is automatically perpendicular to the centre line and generates 90° at the point of where the lines intersect.



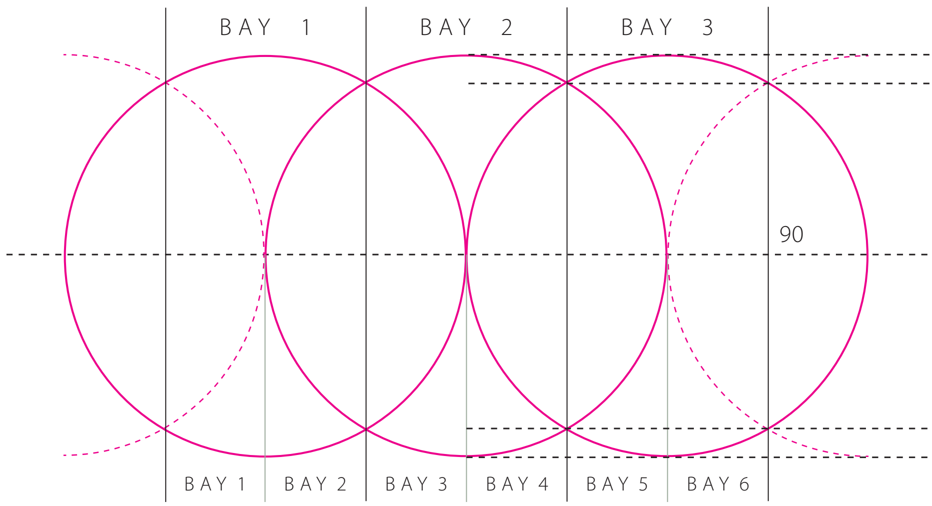
3 Drawing a precision perpendicular

A precision perpendicular is a line drawn at 90° to another at a precisely predetermined point. Step 1 is to determine the position of the point by drawing a straight horizontal line and marking the point on it (here by an arrow). Step 2 is to draw a circle from the marked point so that it cuts the centre line in two further points (the length of radius is a matter of choice). Step 3 is to draw a vesica from the two points where the circle cuts the centre line. In the final step, bisection of the vesica gives the perpendicular.



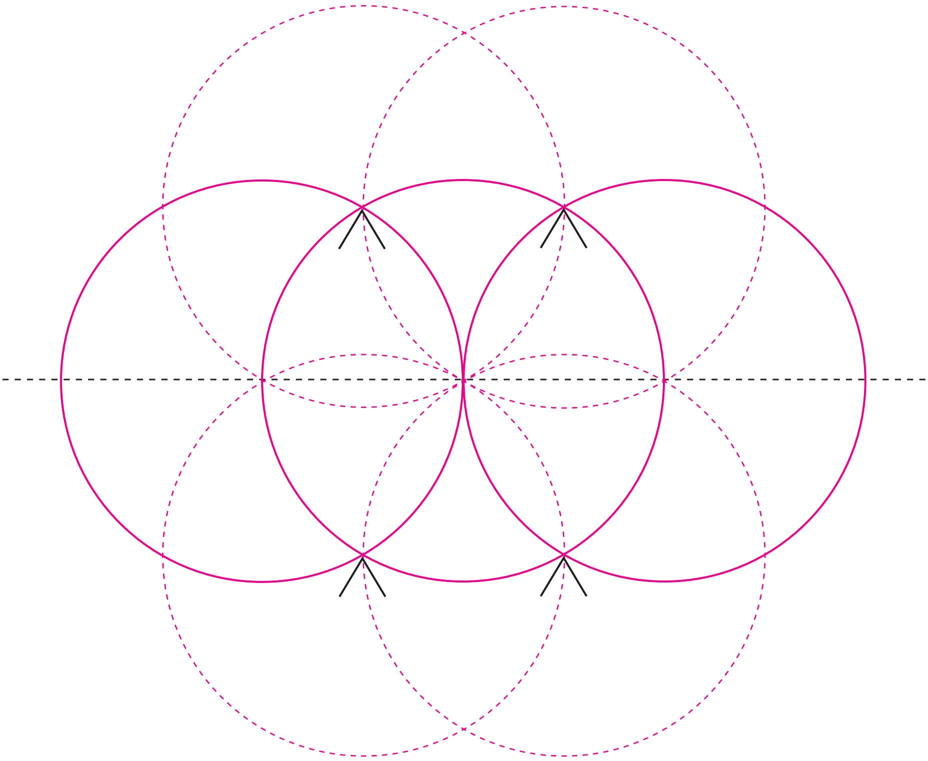
4 Drawing a short cut precision perpendicular

Follow the procedure for a precision perpendicular except that instead of drawing the full circle and arcs of the vesica, scribe just the vital intersections as shown above. These are then bisected in the same way to give the perpendicular.



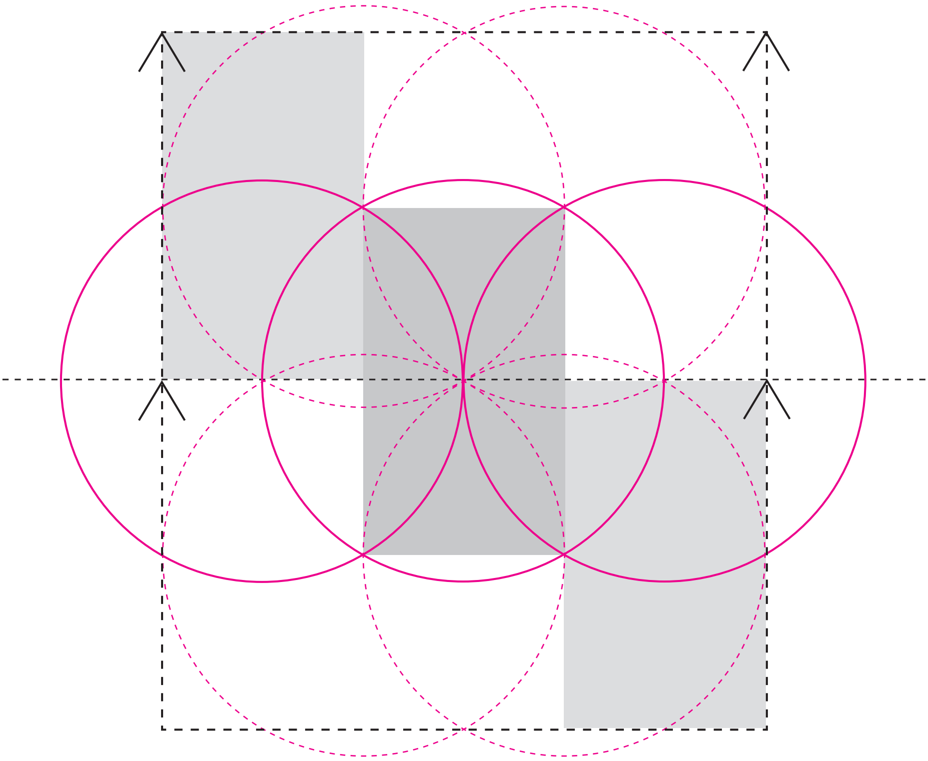
5 Three Circle sequence

Any number of identical radius circles can be drawn along a centre line but a three circle sequence is shown here. It follows that if two identical circles on a centre line generate a perpendicular then every pair of circles will and this means that a circle sequence can generate a cross wall bay rhythm for a linear building. In the drawing above the three bay rhythm results from the vesica intersections but this rhythm can be doubled to six bays if parallels are also drawn where the circle circumferences kiss along the centre line. Parallels to the centre line can be drawn either as tangents to the circle circumferences or through their points of intersection to give long wall alignments as shown on the right of the drawing.



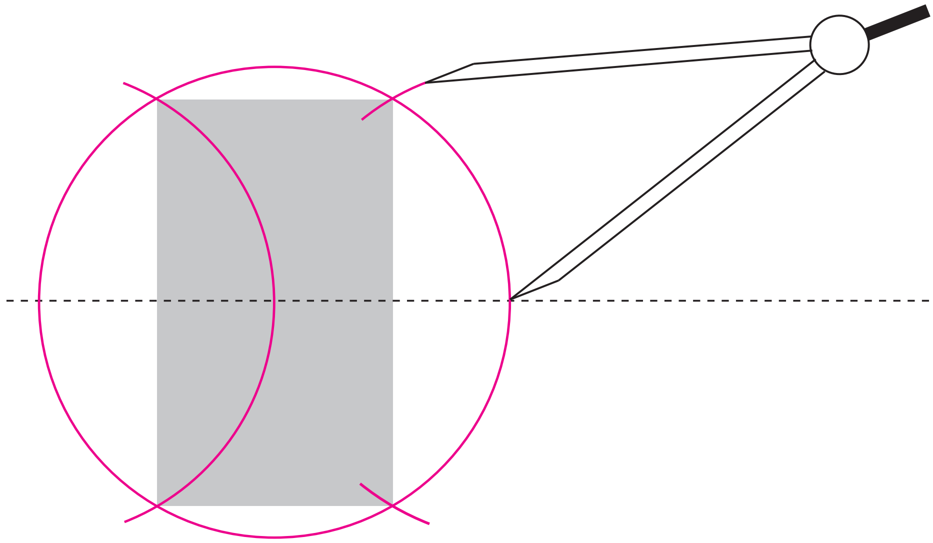
6 Drawing the Daisy Wheel

Although the daisy wheel is commonly drawn as six circles around the circumference of a central circle it is more accurately drawn as a development of the three circle sequence. This is because the relationship of the three circles in the sequence is governed by the centre line and they intersect each other at the four centres needed to complete the construction of the wheel.



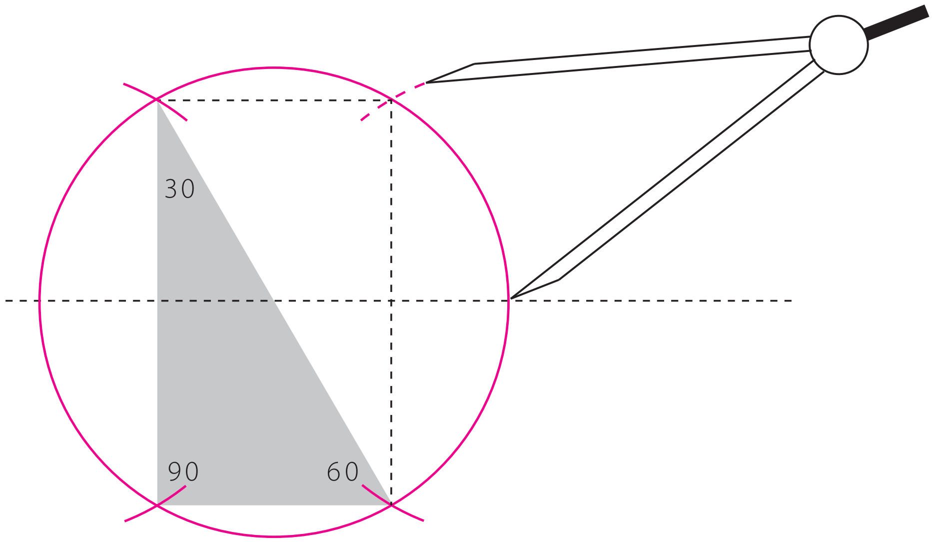
7 The daisy wheel as a source of root 3 rectangles

Despite the fact that the daisy wheel's full construction is composed entirely of circles and that the central circle's well known six-petalled wheel is composed entirely of arcs, all drawn by compass, the construction is a source of rectangular proportions, specifically the root 3 rectangle. A rectangle drawn through all six outer points of intersection (the large dashed rectangle) results in two large horizontal root 3 rectangles (the upper rectangle is indicated by arrows). A rectangle drawn between four of the central wheel's petal tips generates a small vertical root 3 rectangle. It can be seen that the large and small rectangles share a harmonic relationship where each large rectangle equal to three small ones.



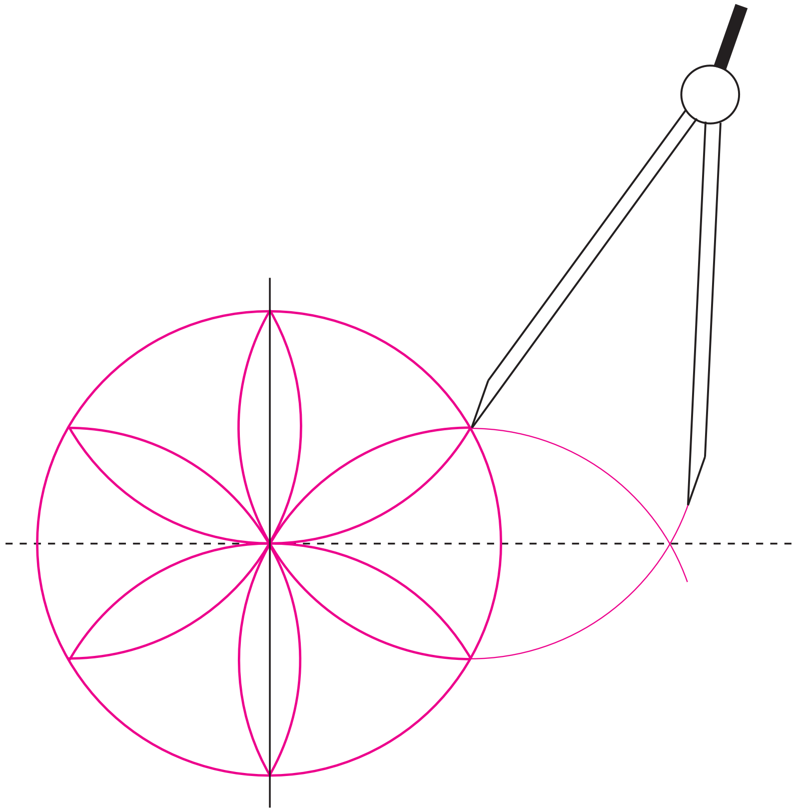
8 Short hand construction of the root 3 rectangle

The circle must be drawn on a centre line so that it is cut from either end of its diameter. With the compass set to the same radius draw arcs from the ends of the diameter so that they each cut the circle at two points. The full arc is shown on the left and the shorthand arc on the right. Connection of the four points gives the root 3 rectangle. The short side of the rectangle is identical to the circle's radius. This is the fastest and most accurate way to draw a root 3 rectangle because the radius is the only dimension needed and the rectangle's right angled corners arise automatically from the compass drawing.



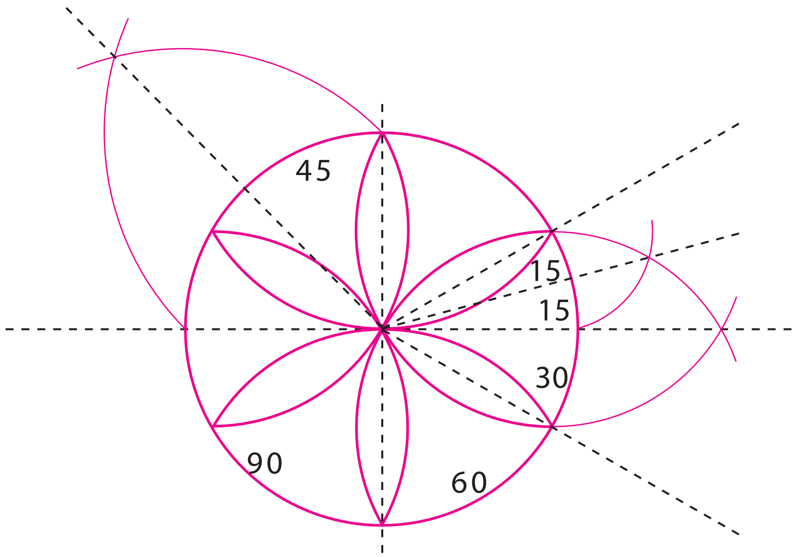
9 Construction of a right angled triangle

The root 3 rectangle can be bisected on its diagonal to give a right angled triangle with angles of 30°, 60° and 90°. Clearly, only three cuts need to be scribed on the circle's circumference, the top right cut being superfluous.



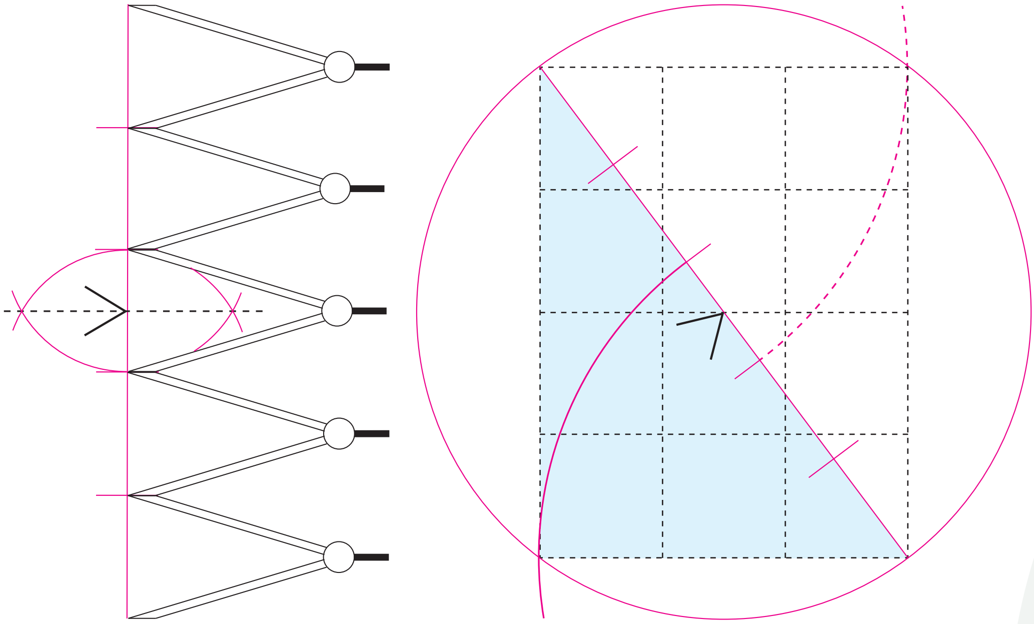
10 Drawing a perpendicular from the daisy wheel

The daisy wheel can be used to generate a perpendicular. In this drawing the wheel is constructed on the vertical line. Extensions of two of its arcs intersect at a point level with the axis and a line drawn through these points gives a horizontal perpendicular. The extended arcs can be drawn at any time, either during the wheel's initial construction or later, from an existing wheel.



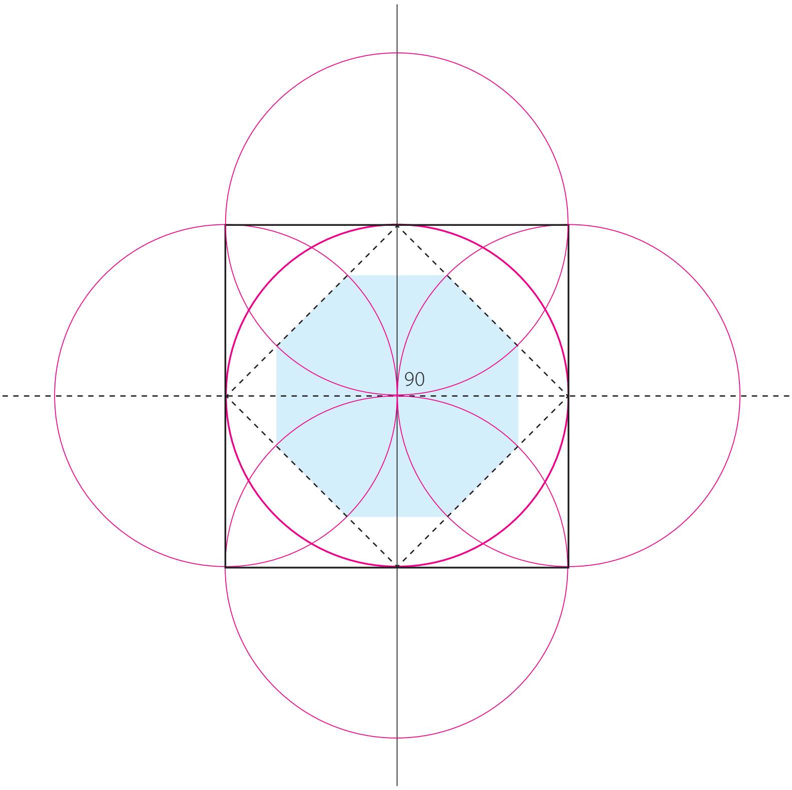
11 Using the daisy wheel as a protractor

The daisy wheel can be used to generate many angles. The wheel's natural division is into six 60° angles, constructed either by drawing diameters across the wheel between the opposite petal tips or connecting each tip to the wheel's axis. If 60° is bisected it gives 30° and this in turn, bisected, gives 15°. The angle between the perpendiculars is 90° which, bisected, gives 45. Further bisections can be made to give smaller angles so that 45° ÷ 2 = 22½° or 15° ÷ 2 = 7½°. Larger angles can be attained by addition so that 90° + 45° = 135° and 90° + 60° = 150° whilst 60° + 30° + 15° = 105°, etc. Because many protractors a manufactured for school use they are too small to project angles accurately to larger scales so the wheel, which can be drawn at any size, is a useful tool for laying out and cutting templates for either regularly used or unusual angles.



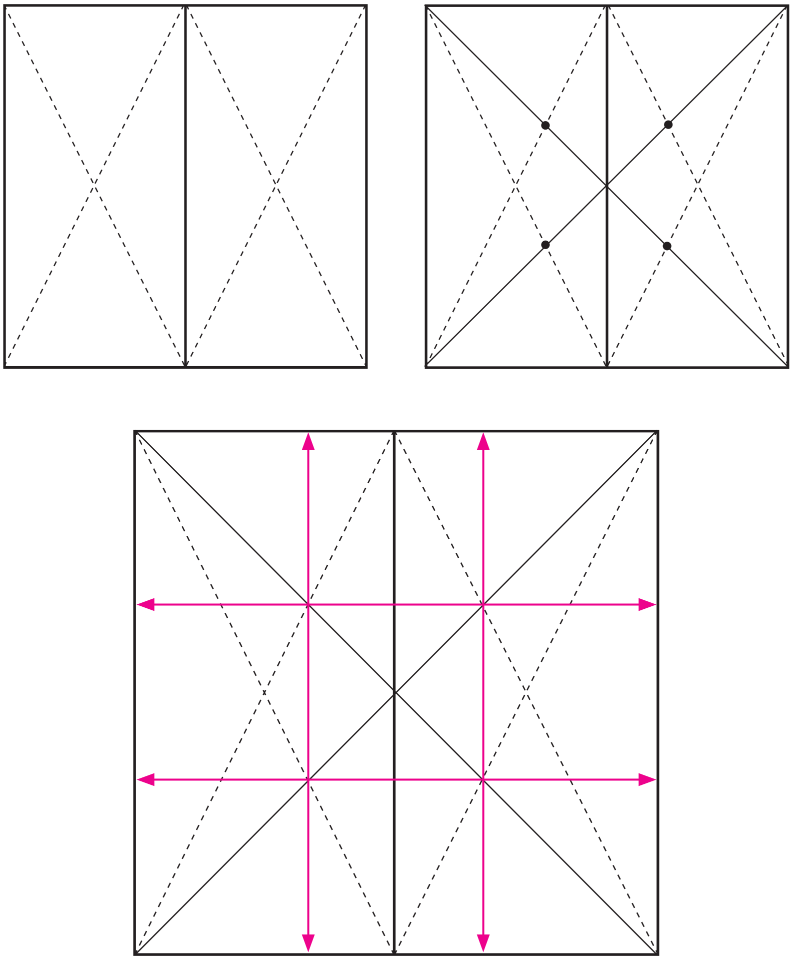
12 Constructing the 3 4 5 triangle by compass

To construct the 3 4 5 triangle using a compass we start from side 5. If sides 3 or 4 have known or intended lengths it is easy to divide by 3 or 4 to determine the unit length for a third or quarter of a side. Once the unit length is known it can be stepped out five times along a line to give the length of side 5. The centre point of line 5 (at 2½ units) is found (by drawing and bisecting a small vesica in the central unit) to give the circle's axis. A circle with a radius of 2½ units is drawn from the axis so that the length of side 5 is automatically the circle's diameter. With the compass set to 3 units an arc is drawn from one end of the diameter until it cuts the circumference. When this point is connected to both ends of the diameter it forms the 3 4 5 triangle (shown in tone). The same construction can be repeated from the opposite end of the diameter to give the full 3 x 4 rectangle (shown in dashed line). The beauty of this construction is that there is no need to use a square to establish the triangle's right angle because the geometry generates it automatically. The 3 4 5 triangle is best remembered from Pythagoras' famous theorem where *the side on the hypotenuse is equal to the sum of the squares on the other two sides*. Side 5 is the hypotenuse (the side opposite the right angle) and its square is 5 x 5 = 25. The other two sides' squares are 3 x 3 = 9 plus 4 x 4 = 16 and, as 9 + 16 = 25, the theorem is proved.



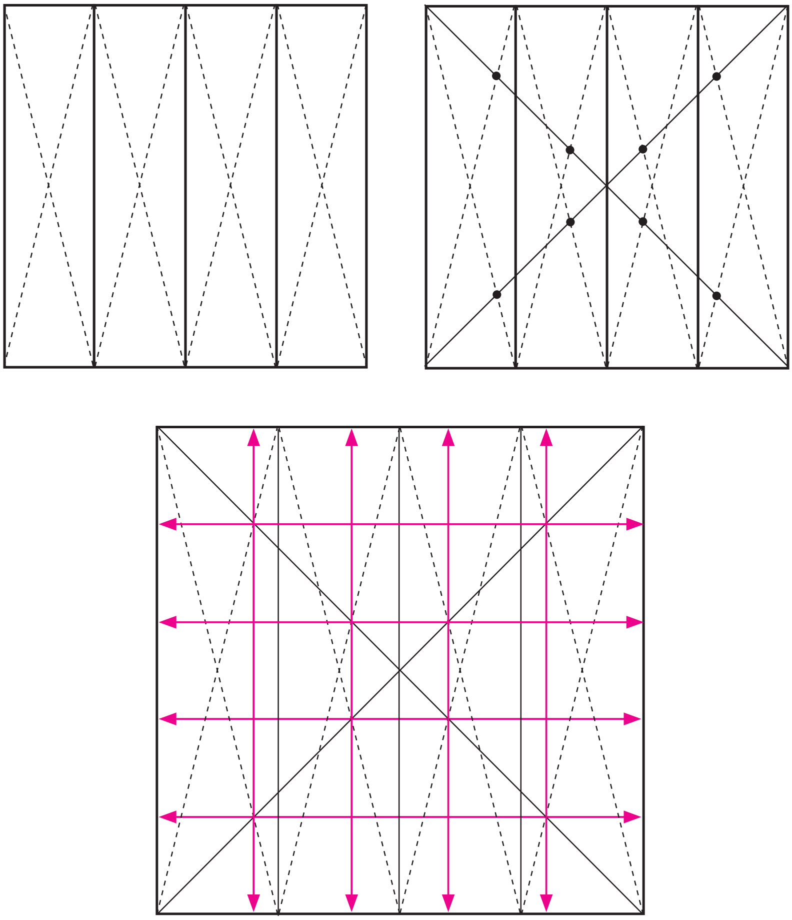
13 Constructing the perfect square by compass

The first step is to draw a centre line and perpendicular (see drawings 2, 3 and 4). The first circle is drawn from the intersection of the lines so that the lines cut its circumference in four equidistant poles. Using the same radius, draw four further circles from the poles. The four outer circles intersect each other at four points and these points are the corners of the square. The drawing method is completely free from the need to construct right angles at the corners of the square. For setting out large squares, radius rods are ideal. These need to be slightly longer than the radius and drilled through at exactly at either end of the radius for trammel pins. The diameter of the central circle is equal to the side of the square. Other polygons can be made from the construction. Connecting the central circle's four poles forms a diamond and, where this cuts the arcs of the other four circles, a perfect octagon is produced. Connecting all the points where the central circle is cut by the outer circle arcs and the perpendiculars generates a twelve-sided figure.



14 Dividing a square into three sectors using diagonals

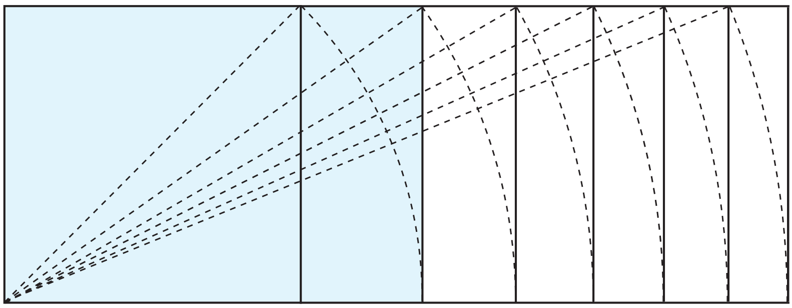
To divide a square into three sectors it must first be divided in half (two sectors). The diagonals of the half squares are drawn first. Then the full diagonals of the square. The full diagonals cut the half diagonals in four places. Vertical and horizontal lines drawn through the four points of intersection divide the square into vertical and horizontal thirds. The vertical and horizontal lines also subdivide the full square into nine equal small squares. This principle can also be applied to rectangles.



15 Dividing a square into five sectors using diagonals

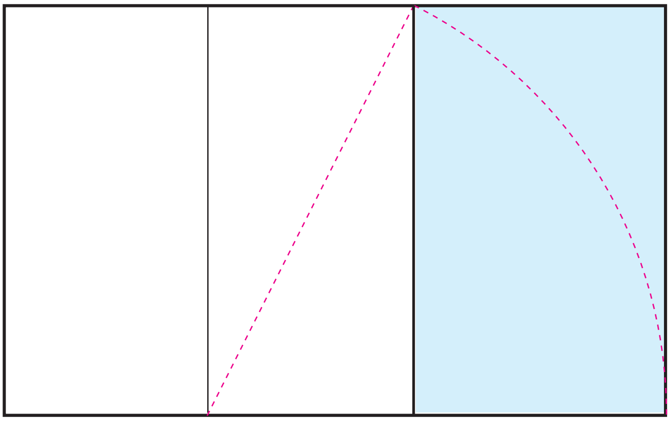
To divide a square into five sectors it must first be divided into quarters (four sectors). The diagonals of the quarters are drawn first. Then the full diagonals of the square. The full diagonals cut the quarter diagonals in eight places. Vertical and horizontal lines drawn through the eight points of intersection divide the square into vertical and horizontal fifths. The vertical and horizontal lines also subdivide the full square into twenty-five equal small squares. This principle works with rectangles.

NOTE This system of division will increase any number of even divisions by one so that halves are converted to thirds, quarters to fifths, sixths to sevenths, etc. To arrive at a division of thirteen the procedure would be to halve to get thirds, halve the thirds to get sixths, halve the sixths to get twelfths, draw diagonals in each twelfth and then the full diagonals. Lines through the points of intersection will give thirteen vertical and horizontal divisions.



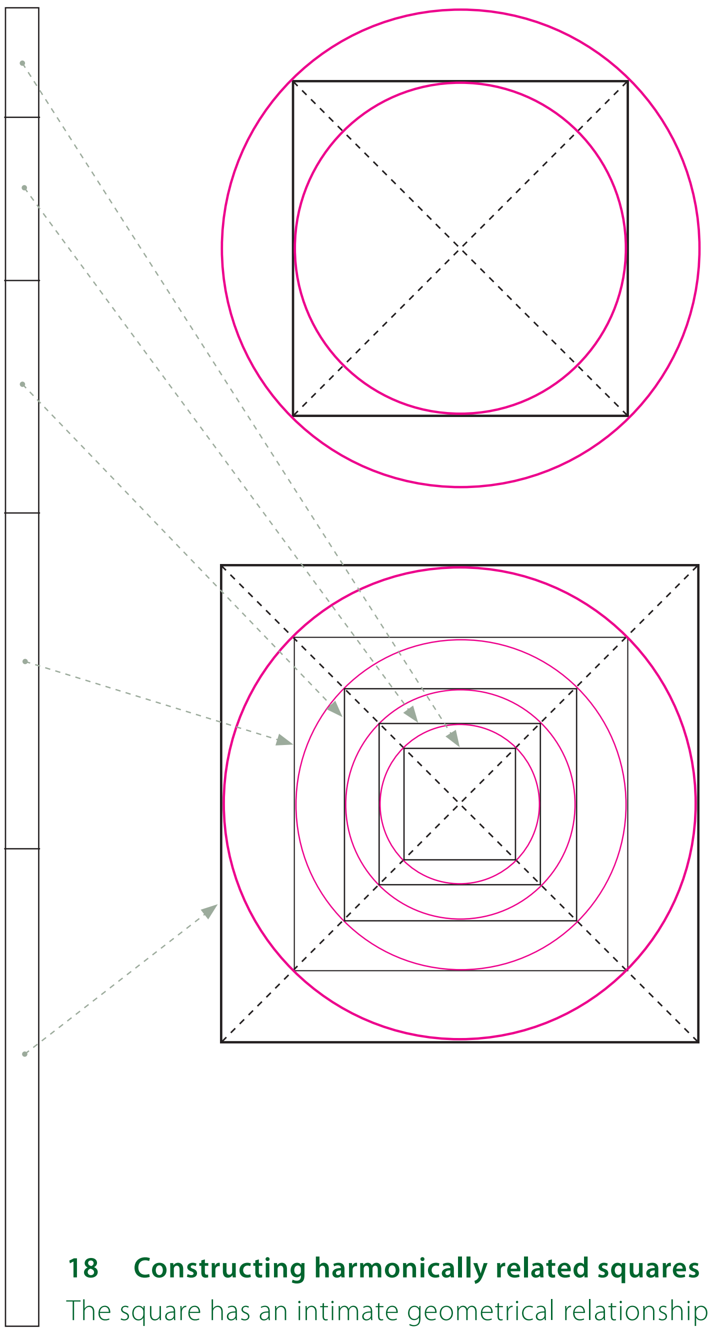
16 Constructing root 2, 3, 4, 5, 6 and root 7 rectangles

The rectangles are a harmonically related sequence where each consecutive rectangle's diagonal is transmitted by compass arc down to the base line to define the next rectangle's boundary. The sequence usually begins with the square so that the square's diagonal projected to the base line establishes the root 2 rectangle, the root 2's diagonal establishes the root 3 and so on. The sequence can theoretically evolve to eternity but comes to a halt when drawing the diminishing sectors becomes impossible. The related rectangles are useful in establishing floor, wall, window and door proportions that have harmonic resonance. The root 2 rectangle owes its name to the fact that a square with sides of 1unit in length will have a diagonal of 1.4142 (the square root of 2) which becomes the rectangle's long side. Most of the root numbers, like root 2, are difficult to measure or calculate but can be drawn easily by a child with a straight edge and compass. The root 3 rectangle can be drawn more easily by compass, see drawing 8.



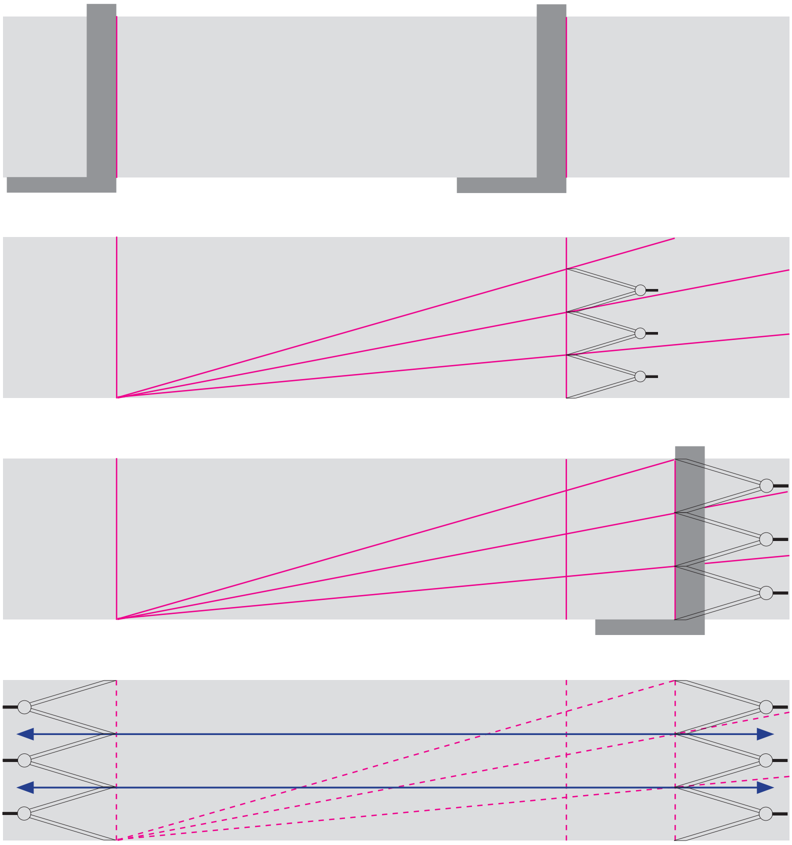
17 Constructing the golden rectangle

The golden rectangle is developed from the square which is first divided into two halves. The diagonal of half the square is transmitted by compass arc down to the base line and the square is extended to this point to form the golden rectangle. The extension alone (shown as a blue tone) is also a golden rectangle and, with its long side equal to the large rectangle's short side, the two rectangles share a harmonic proportional relationship. The golden rectangle was used in Classical and renaissance times as a source of proportion for building plans, facades, doors and windows.



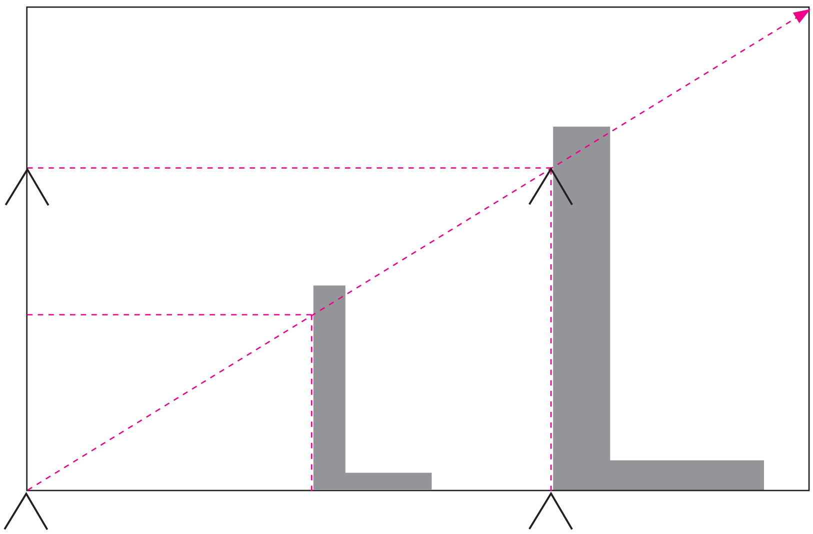
18 Constructing harmonically related squares

The square has an intimate geometrical relationship with two circles, the larger external circle passing through the square's corners and the smaller internal circle kissing the square at the centre of each side. This relationship can be extended indefinitely by the construction of ever larger or ever smaller squares that maintain a constant proportional relationship to each other. The rule shows the proportional relationship of the five squares' sides. The rule can be used, for example, to establish harmonically related widths or heights for different sized windows.



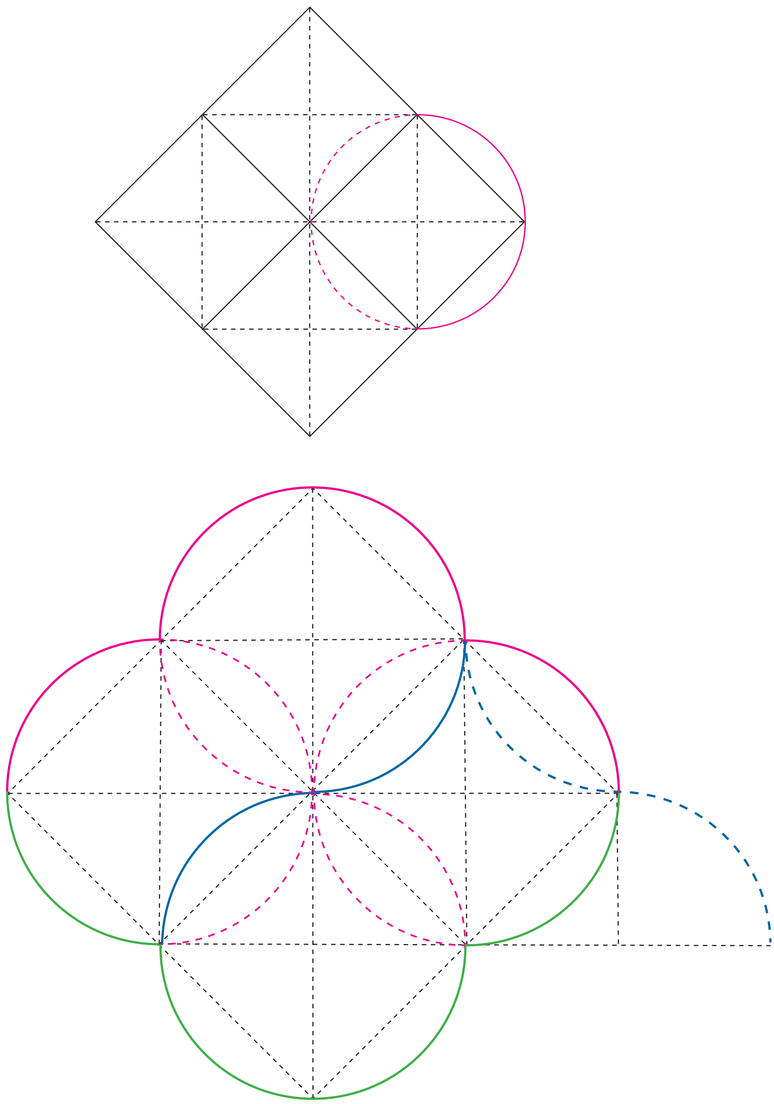
19 Dividing timbers into odd numbered equal widths

Scribe two right angles across the timber (there are no exact positions). If the timber is to be divided into three widths set the dividers (by rule of thumb) so that three steps on the right hand scribed line are less than the full timber width. Mark the three points on the line. Snap chalk lines between the left scribed line (where it meets the timber's lower edge) and the three points on the right scribed line so that they continue towards the timber's end. Scribe a new right angle where the highest chalk line crosses the timber's upper edge. Where this right angle cuts the chalk lines take a new divider reading and step it out on a right angle at either end of the timber. Snap new chalk lines through the steps parallel to the timber. The method is useful either where the number of divisions is otherwise difficult, such as eleven, or the mathematics of dividing a 19 3/16 inch timber into seven would cause brain haemorrhage.



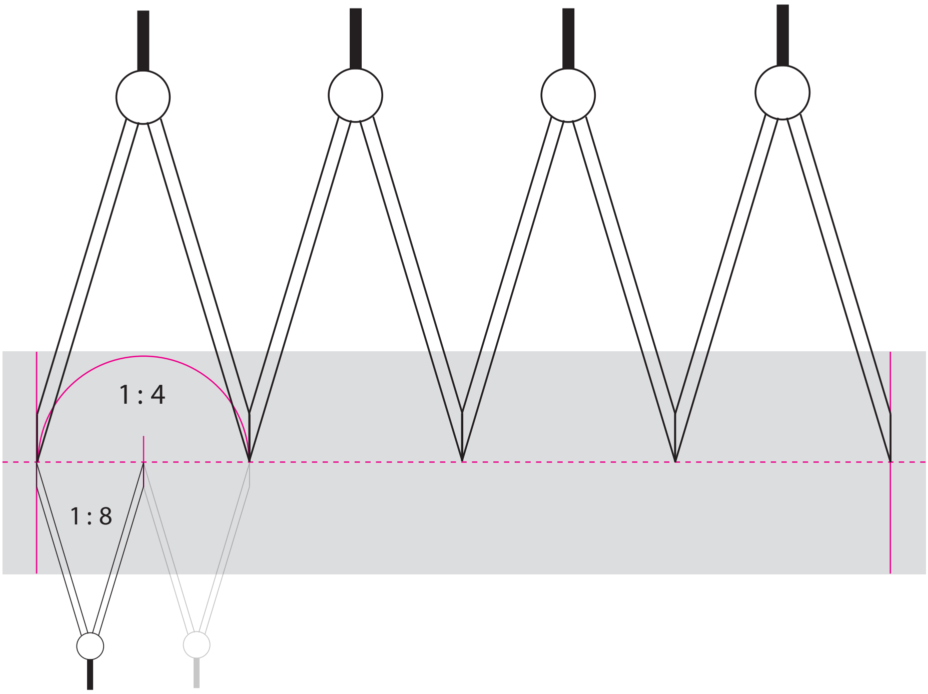
20 Constructing proportionally related rectangles

It is easy to reproduce the exact proportions of a rectangle at a different scale. Draw or chalk line a diagonal across the existing rectangle and then place a carpenter's square against its base line the required distance from the diagonal's origin. Where the carpenter's square cuts the diagonal, scribe a vertical along the square's edge. Take the length of this line, mark it on the left edge of the rectangle and connect the marks to construct a parallel to the base line. The process can be repeated wherever the square is placed against the diagonal and the rectangle can be of any proportion, either horizontal or vertical (the rectangle shown is a golden rectangle). It is a useful technique for maintaining proportional relationships between windows of different sizes within a single building, especially where modern windows are being installed in a historic building (by taking the proportions of an existing window and scaling up or down).



21 Constructing a quatrefoil, tripartite arch and ogival arch

The construction starts from a square which is subdivided into four quarters or small squares. Diagonals are drawn in each quarter and circles are drawn from where they intersect so that they pass through the small squares' corners. The drawing is revolved through 45° so that when the quatrefoil is drawn it is in its correct position with vertical and horizontal centre lines. If the quatrefoil is cut along its horozontal centre line it gives a tripartite arch with a half-circle head and quarter-circle shoulders. The ogival curve (a curve with equal convex and concave sectors) follows the quarter- arcs of two adjacent circles. The curve is duplicated in mirror image to form the ogival arch. This can be constructed by drawing more squares, diagonals and circles or, more simply, by tracing the first curve.



22 Stepping out

Stepping out with dividers along a chalk line is probably the most accurate way to translate dimensions from drawings onto timbers. Dividers can be set to any dimension irrespective of its numerical value, which eliminates the headache of numbers, and they record the dimension between two needle fine points that, unlike steel tapes, do not twist or flex. The only opportunity for error is in remembering the number of steps to be taken. This risk can be halved by doubling the initial divider dimension to give half the number of steps. For example, if a drawing is at 1:24 scale, doubling the initial divider dimension reduces the steps to 12. The drawing shows an initial divider dimension taken from a 1 : 8 scale drawing and doubled to give just four steps along the chalk line. The simplest way to double the initial divider dimension is to scribe a half circle on the chalk line which doubles a radius into a diameter.

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