

GEOMETRICAL DESIGN
at TRIANGULAR LODGE



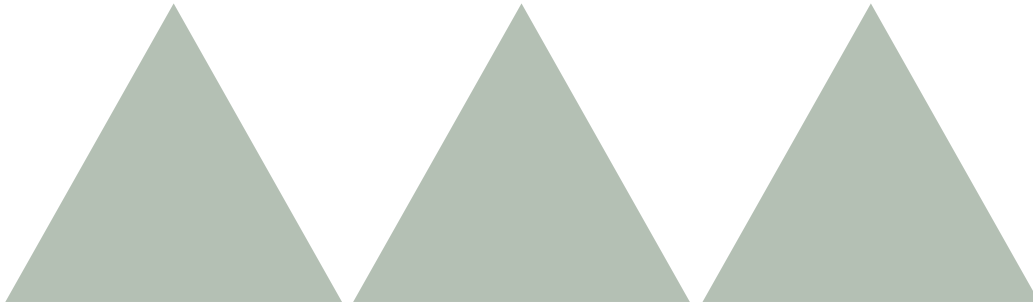
Laurie SMITH
HISTORIC BUILDING GEOMETRY

Laurie Smith is an independent early-building design researcher, specialising in geometrical design systems. Because geometry was present in the medieval curriculum he uses geometrical analysis to excavate the design methodologies of the time, a process he thinks of as design archaeology. He lectures, writes and runs practical workshops on geometrical design.

HISTORIC BUILDING GEOMETRY is his personal website.

The three traditional tools used for hand drawn geometrical construction were dividers, straight edge and scribe, historic precursors of the modern compass, ruler and pen. The straight edge, a ruler without measurements, served only as a guide for drawing straight lines. The geometrical constructions in this pamphlet are described as if drawn manually using a compass, straight edge and pen on pure white paper. The reader is encouraged to draw the constructions because drawing is the best route to understanding.

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Triangular Lodge

The genesis and evolution of
Thomas Tresham's geometrical masterpiece -

a geometrical analysis and design reconstruction

HISTORIC
BUILDING
GEOMETRY

Laurie
SMITH



I N D E X

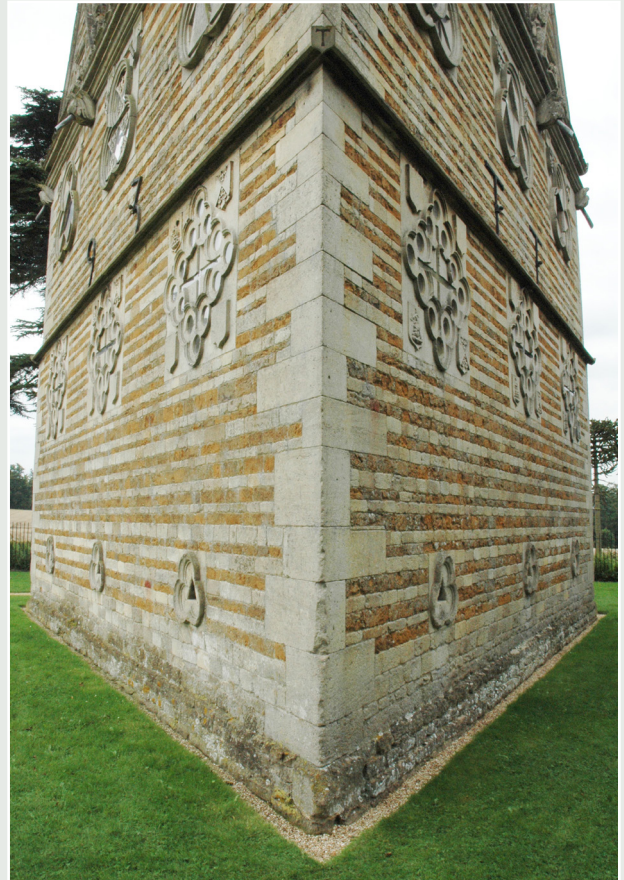
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Introduction

Triangular Lodge, designed geometrically by Sir Thomas Tresham and built on his estate at Rushton in Northamptonshire between 1593 and 1597 as a lodge for his Warrener, is a unique architectural gem. Though diminutive in scale the building is a striking visual play on the French 3 (tres) in Tresham's name and, simultaneously, the devout expression of his Christian beliefs through symbolic numerical and visual representations of the Trinity, predominantly through equilateral triangles, trefoils and whirling squares composed of trefoils and crosses.

Triangular Lodge is framed between quoins of pale oolitic limestone and built from alternating courses of the same stone and a softer yellow ochre ironstone. The oolitic limestone, a fine mason's stone, even in texture and perfect for sculptural work, is used for the precision geometrical layout of the windows and the entablature's cut lettering. The ironstone, which erodes much more rapidly than oolitic limestone, is subordinate and used solely for the contrast of its wonderful colour and texture.

Tresham's theological symbolism, letter-cut as enigmatic wording and imagery into the stonework of Triangular Lodge, has been written about extensively elsewhere¹. The focus here is on his geometrical design methods: following his path from the germination of a triangular seed, through its nurture and evolution, to the eventual blossoming of a delicate and exquisite architectural flower.



Triangular Lodge north and south-west facades



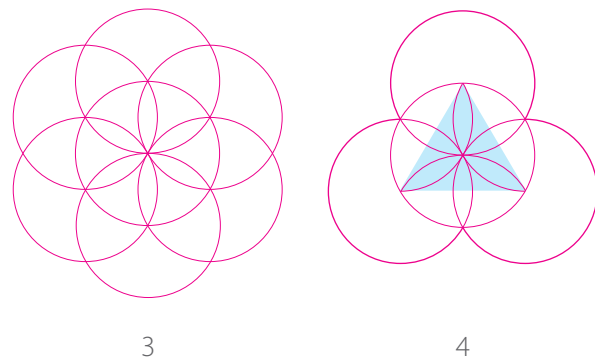
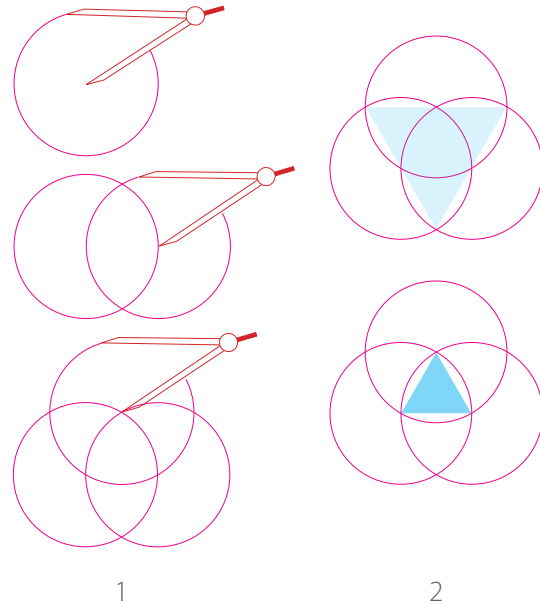
Triangular Lodge north facade

Tresham's trefoil and equilateral triangle

Tresham's innovative conceit was to combine his own name with the Trinity in a building that was an equilateral triangle on plan, a concept that almost certainly arose from the compass construction of his trefoil logo. As a geometer, he would have known that the trefoil could be drawn simply from three equal radius circles, the second drawn from the circumference of the first, the third from the intersection of the first and second, a drawing that automatically has perfect symmetry, figure 1. Significantly, the trefoil's three circles intersect at six points, three at the perimeter of the construction and three at the centre, both groups yielding equilateral triangles if connected, figure 2. The three sides of the large equilateral are simultaneously diameters of the trefoil's three circles while the sides of the small equilateral are the trefoil's radii. The trefoil and equilateral triangle therefore share a symbiotic geometrical symmetry where secret equilaterals remain concealed within the trefoil unless revealed through the act of drawing.

The trefoil and equilateral triangle can also be drawn from the daisy wheel, constructed from six circles drawn around the circumference of the seventh to produce the familiar central daisy pattern, figure 3. Alternate outer circles form a trefoil while linking three alternate petal tips generates one of several concealed equilateral triangles, figure 4.

The daisy wheel is also part of a continuum of interlacing arcs that extend to infinity



An equilateral triangle can be drawn at any scale so Tresham had to decide on the dimensions of his equilateral floor plan. He chose 33 feet for the length of the walls, a number with clear theological resonance but also a double Rod, a single Rod being $16\frac{1}{2}$ feet. The double Rod is the first whole number in a geometrical progression of Imperial fractions, each of which is half of the next greater and double the next smaller.

$1\frac{1}{32}$ $2\frac{1}{16}$ $4\frac{1}{8}$ $8\frac{1}{4}$ $16\frac{1}{2}$ 33

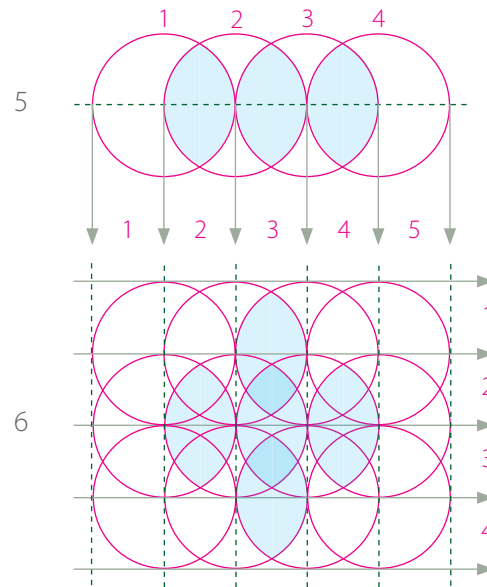
Because a circle's radius is half its diameter and its diameter twice its radius, halving and doubling is easily attained practically using a compass or dividers so that, for example, a $16\frac{1}{2}$ inch radius doubles to a 33 inch diameter or vice versa. 33 inches is in 1:12 ratio to 33 feet (*12 inches = 1 foot*) so it follows that $16\frac{1}{2}$ inches is in 1:24 ratio. Any divider reading taken from Tresham's design could be doubled and then stepped out for twelve paces with dividers along a string or chalk line to give the full scale dimension.

Wall and gable proportions

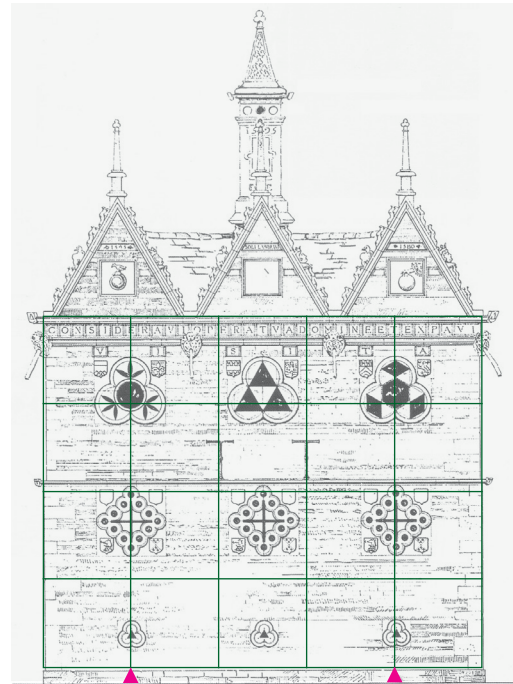
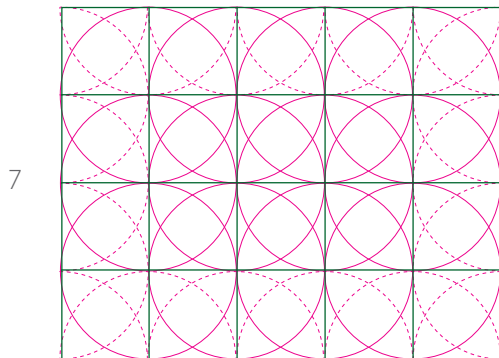
Measurements reveal a ratio of 4:5 between the plinth and entablature, which could be attained through ruler dimensions and set square projection to form a grid of equal squares, but this is a dimension driven process and, as a geometer, Tresham would have known a swifter and easier compass-based method. All he needed was a radius: one fifth of the 33 feet wall length at $6\frac{3}{5}$ feet or $6\frac{3}{5}$

Imperial dimensions are given because these were the original design dimensions and the numerical concepts are therefore clearer.

inches at 1:12 scale (or $3\frac{3}{10}$ inches at 1:24). It is a geometrical paradox that a four circle sequence drawn on a centre line generates five equal divisions and, more importantly for Tresham's symbolic aims, the four circles embody three secret vesicas² so that, in drawing four circles, Tresham was also expressing the Trinity, figure 5. In the same way that four circles generate five divisions, three circles generate four divisions. Tripling the four circles generates a more complex geometrical grid that embodies a tripling of the vesicas but also enables them to be conceptualised in other, secret formations such as three horizontal and three vertical vesicas that combine in a five vesica cross, figure 6.

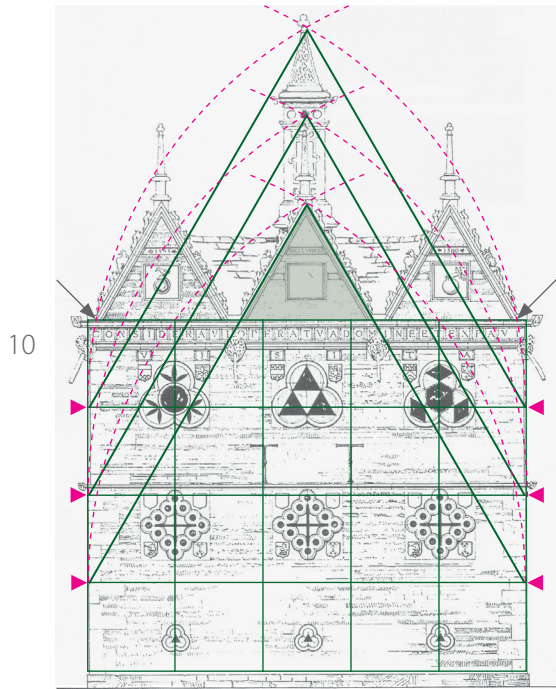


The compass-drawn circle grid is completed by drawing quarter-circle arcs, half-circle arcs and, finally, tangents to generate the square grid, figure 7. A simple example of a compass grid can be found within the monastic floor tiles of Winchester Cathedral where a selected three by four circle grid also gives a four by five grid of squares, figure 8. The grid, which is swift to draw by repeating the common radius of the first circle, functions exactly like a modern graph paper.

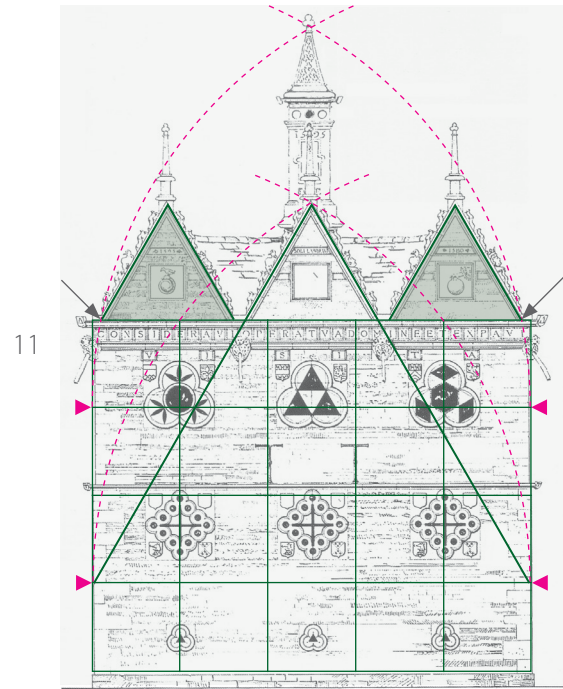


It can be seen, in figure 9, how the four by five square grid determines the external proportions of the wall and gives the centres, arrowed, for the vertical alignment of the outer windows. However, a circle grid offers more points of intersection than a square grid and, more importantly, these points of intersection give incommensurable locations that cannot be found within a square grid.

Combining the two elements of the design, the equilateral triangle and the four by five



grid, enables the remaining major elements of the facade to be constructed, figure 10. The method is simple: an equilateral of floor plan proportion is drawn from each of the three arrowed levels, between ground level and the entablature, using compass arcs from either end of each level that intersect at the apex of the equilateral. The lowest equilateral defines the profile of the central equilateral gable, the second defines the central opening in the chimney, the third defines the apex of the chimney's pinnacle, which terminates

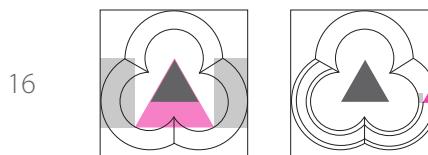
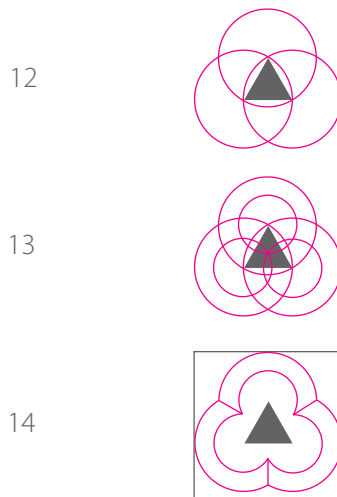


in a Tresham trefoil finial. The outer gables, which are identical to the central gable and have their equilateral triangulation drawn from it, have their outer placement exactly where the highest equilateral's construction arcs cut the entablature, the points arrowed in grey in figures 10 and 11. The fact that these arcs cut the entablature at the precise points where the gable triangulations commence is eloquent confirmation that geometrical analysis as far as this stage mirrors Tresham's own geometrical development.

Fenestration: the basement trefoils

Tresham's fenestration expresses the Trinity in triplicate: in the applied geometrical pattern of three windows on three floors on three walls. The basement and upper floor windows are based on his personal trefoil, those at the basement level being small, simple and hardly more than ventilation while those on the upper floor are large and geometrically complex. Where the nine basement trefoils have identical geometrical structure, the nine upper floor trefoils express a varied vocabulary of geometrical explorations. Sandwiched between the basement and upper floors, the main floor's fenestration has nine identical windows designed using whirling square geometry. In rising from the ground to the entablature the geometry passes from simple through to complex and, as it does so, from compass to square to compass in a classic juxtaposition of circularity and angularity, an interplay between compass-drawn and angular geometrical constructions.

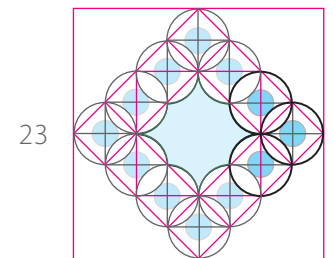
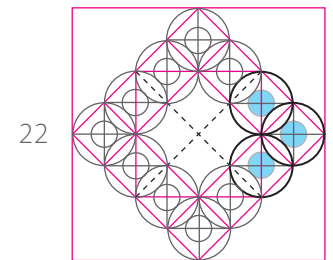
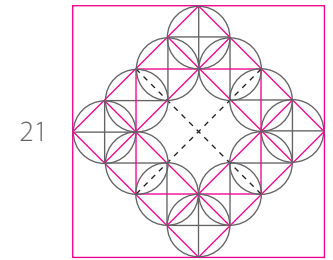
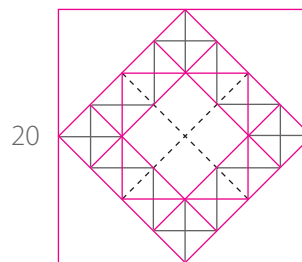
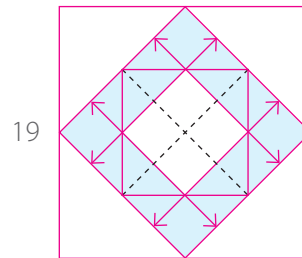
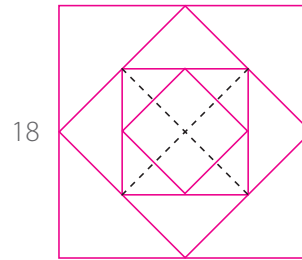
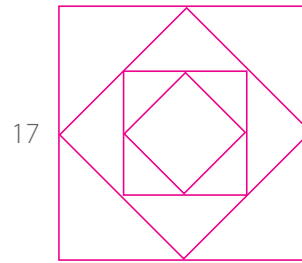
The basement windows commence with an equilateral triangle drawn between a trefoil's three internal points of intersection, figure 12, the equilateral forming the window opening. Three smaller circles, drawn from the angles of the triangle to pass through its centre, generate a smaller trefoil, figure 13. The space between the two trefoils gives the bandwidth of the trefoil's raised surround which, in turn, is bounded by a square, figure 14, and cut in stone in figure 15. Figure 16 shows the trefoil's bandwidth geometry.



Fenestration: the main floor whirling squares

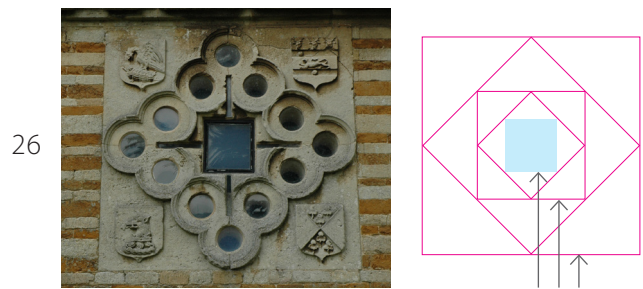
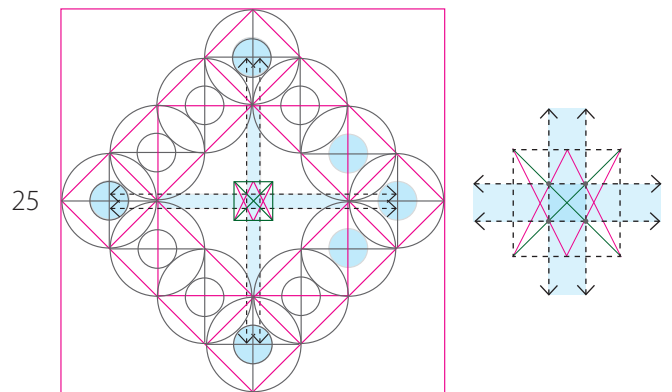
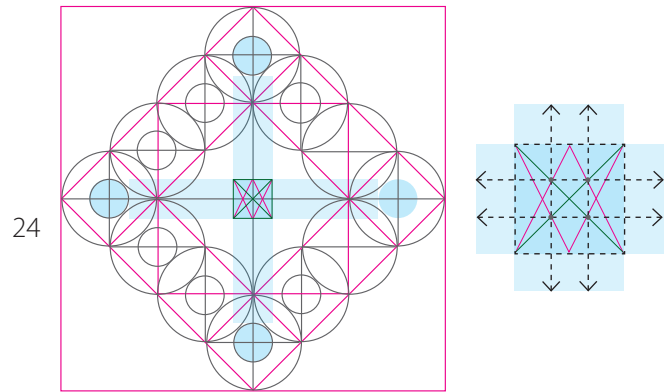
The main floor's fenestration is designed using whirling squares geometry. The whirling squares, figure 17, are a series of harmonically related squares, with every alternative square set diamond-wise in relation to its greater and smaller neighbours. It would have appealed to Tresham for two reasons: the progression can expand or diminish to infinity and is therefore symbolic of eternity while the spaces formed between the squares and their diamond neighbours are triangles, symbolic of the Trinity.

Once the location and scale of the whirling squares are resolved the first stage of the design can commence. Diagonals are drawn in the smallest whirling square, figure 18 and the corners of the smallest diamond are extended to meet the large diamond, figure 19. The construction in figures 18 and 19 forms twelve small diamonds, shown in blue tone, around the large diamond's perimeter. The diagonals of the twelve small diamonds, figure 20, intersect at the axes of twelve circles that are drawn through the corners of the diamonds, figure 21. Finally, twelve smaller circles are drawn from the same axes to define the four sets of trefoil windows (one set is shown in blue tone) in figure 22. The small circles are drawn so that they kiss the circumferences of the adjacent larger circles which frame the boundary of the cusped area at the centre of the window, shown in blue tone in figure 23. It is clear that in Tresham's mind the twelve small circles represent four trefoils.



In the penultimate stage of the design a cross is drawn across the cusped area between the four polar small circles, with the bandwidth of its arms equal to the circles' diameters. Where the arms of the cross intersect they form a small square, figure 24. Diagonals of the square (drawn in green) and diagonals of adjacent half squares (drawn in magenta) intersect at four cardinal points. Horizontals and verticals drawn through the cardinal points divide the square into equal thirds in both directions (see enlargement). Figure 25 shows that only the central third of the band remains visible as the cruciform glazing so that, in conjunction with the invisible two thirds, Tresham could contemplate another secret Trinity.

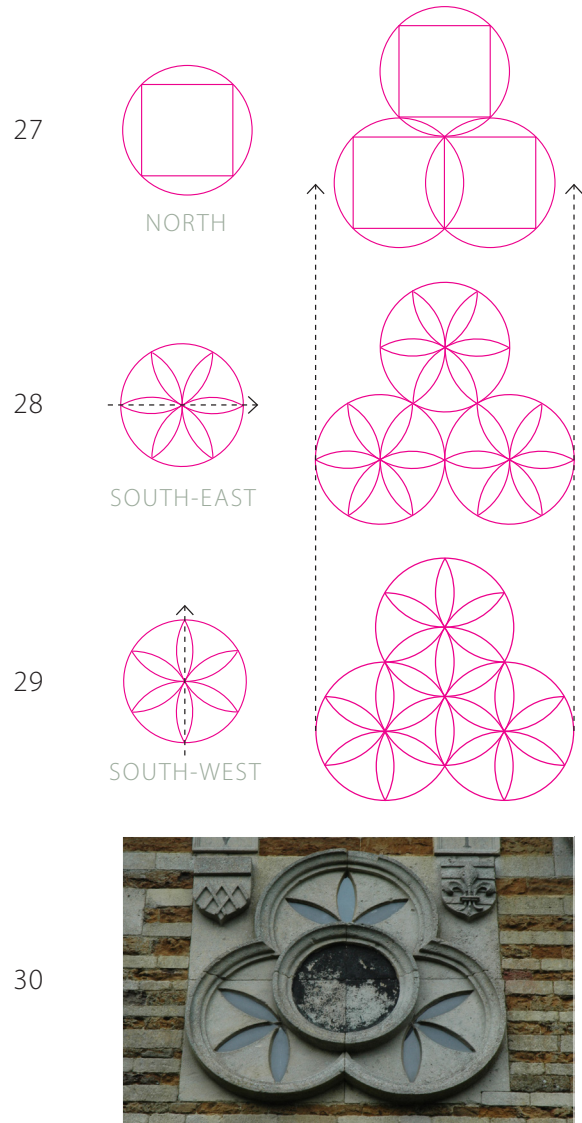
Eight of the main floor's nine whirling square windows have identical glazing but the central window of the south-east facade, to the left of the building's entrance, has an additional square window at the centre of the cross, figure 18. There is a school of thought that this square element of the glazing was cut at a later date although this is a theory that might be difficult to prove. A more appealing alternative is that the window is a further whirling square in Tresham's geometrical scheme, the smallest of the three vertical squares arrowed in figure 26, and in this way yet another expression of the Trinity. Further, as the central window of the central floor it is at the axis of a cross formed from three vertical and three horizontal windows.

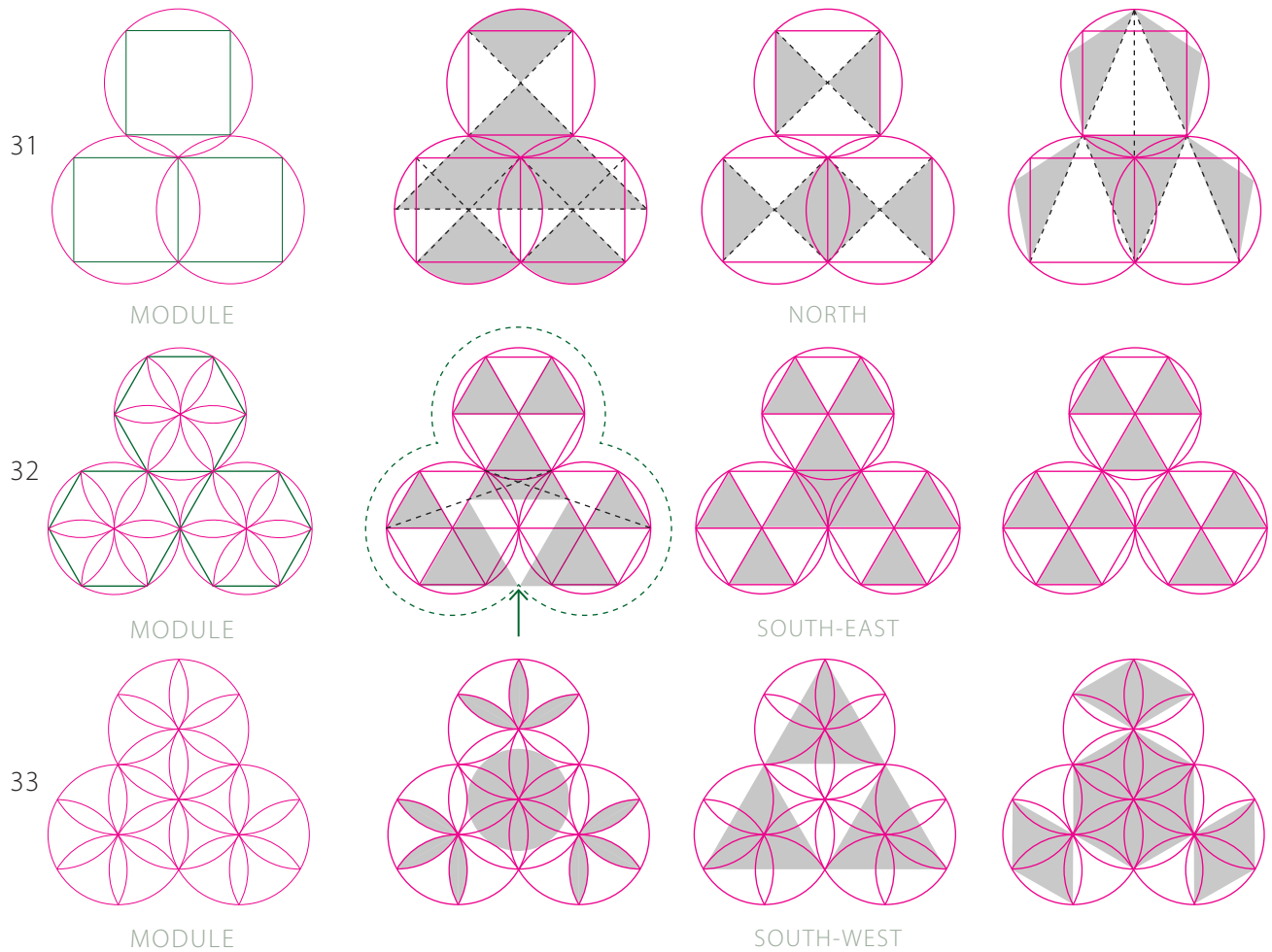


Fenestration: the upper floor trefoils

The fenestration of the two lower floors, where all three windows on all three facades are identical, changes radically on the upper floor where each of the nine windows surrounding the building has a different pattern of glazing, set within the boundary of a trefoil. However, Tresham used a different geometrical construction for the trefoils on each facade and these, if drawn to identical height, are in two different widths. The narrowest is on the northern facade, constructed from a module of a circle surrounding a square, figure 27. The south-east facade's trefoil is constructed from daisy wheels on a horizontal axis, figure 28, while the south-west facade is from daisy wheels on a vertical axis, figure 29. The trefoils in the south-east and south-west facades are identical in width.

The windows explore variations on the theme of triangulation within each trefoil, bringing together the two founding elements of Tresham's symbolic design. But there is one exception to the theme of triangulation and that is the window at the northern end of the south-west facade which directly expresses the daisy wheel compass geometry within the trefoil, figures 29 and 30. Because the circles in the grid overlap each other it follows that each wheel in the grid shares parts of its construction with the adjacent wheels. The trefoil combines the central daisy wheel with three foils from the surrounding wheels, each of which embodies three vesica petals that are expressed as glass in one of the windows.





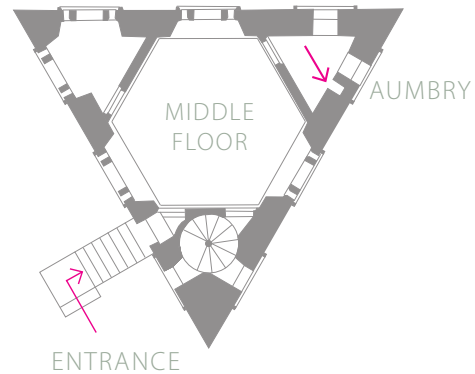
Tresham's three trefoil modules and the three windows designed from each, with additional construction lines in dashed black line and glazing shown in grey tone. Figure 31 shows the north facade, figure 32 the south-east and figure 33 the south-west facade. It is noticeable that in the left trefoil on the south-east facade the two large lower triangles extend beyond the module's boundary (arrowed) but remain within the trefoil's outer boundary (dashed line).

Tresham's esoteric entrance canopy geometry

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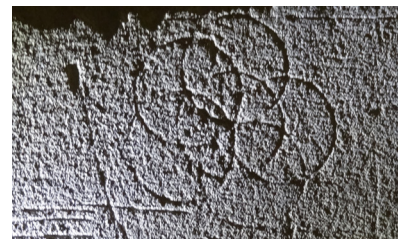


Above the entrance to Triangular Lodge is an unusually shaped canopy that is visual rather than functional and which bears the Latin inscription *Tres · Testimonium · Dant*, meaning three (or *Tresham*) bears witness, figure 34. The entrance leads into the hexagonal main room on the building's middle floor, a room with three triangular closets at the angles of the equilateral floor plan, one enclosing a spiral stair . In the closet furthest from the entrance there is an aumbry recessed into the wall, figure 35, and beneath it there is a geometrical symbol scribed into the stone, figure 36. The aumbry and inscribed stone both have geometrically defined proportions and the symbol itself is drawn at a geometrically defined location on the face of the stone. The symbol's construction is a central circle with five identical radius circles drawn from its circumference, figure 37, and it is from this geometry that the unusual form of the door head canopy can be developed.

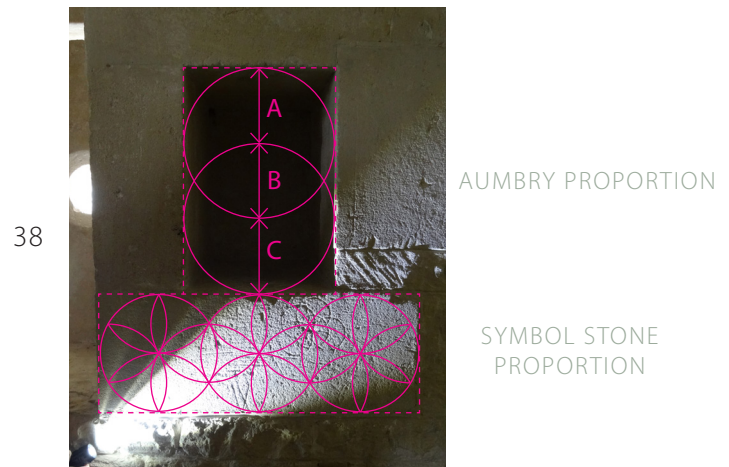
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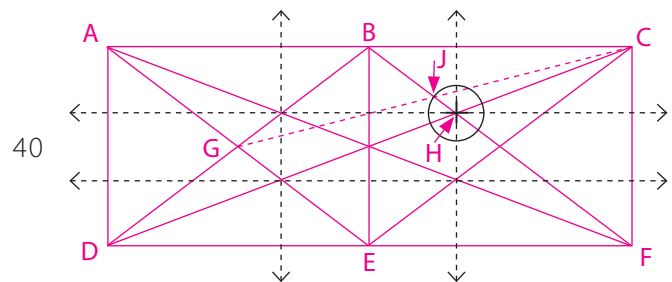
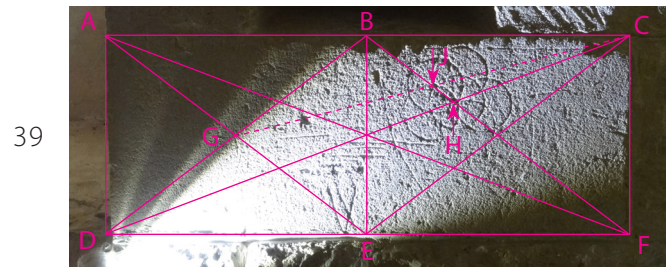
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The following drawings show the geometrical constructions of the aumbry and the inscribed stone. The open space of the aumbry and the solid stone beneath it are both proportioned using compass geometry that expresses the number three (or Trinity). The aumbry is a simple vertical two-circle sequence where the overlapping circles form a vesica piscis, a construction that has three radii (A B and C) along its centre line, figure 38 (aumbry geometry). The inscribed stone's perimeter results from a triple daisy wheel sequence where the wheels are connected at the vesica petal tips, figure 38 (symbol geometry). Tangents drawn around the compass geometries (in dashed magenta line) define the rectangular boundaries of the aumbry and inscribed stone.



The inscribed symbol also has a specific geometrical location on the surface of the stone. In figure 39 the full diagonals AF and DC and half diagonals AE DB EC and BF are drawn first⁴. Lines drawn through the intersections of the full and half diagonals divide the stone into three vertical and three horizontal sectors, the divisions into thirds are shown by dashed black lines in figure 40. The full diagonal DC and half diagonal BF intersect at H, the axis of the symbol's central circle, which is exactly one third down from the top of the stone and one third in from its right hand side, two further expressions of Tresham's consistent theme. A line from C to the intersection of the diagonals at G intersects



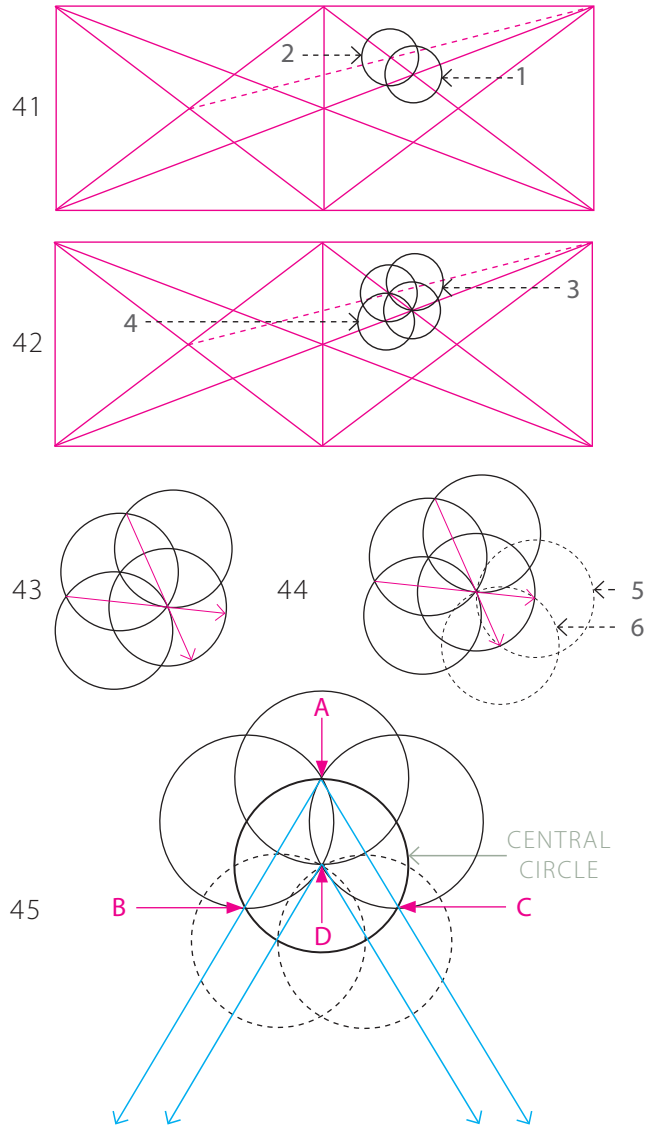
BF at J on the circumference of the symbol's central circle. The distance between H and J is the central circle's radius.

To scribe the symbol, the dividers are placed at H and the central circle drawn through J. The second circle is drawn in reverse with the dividers set on J and the circle drawn through H, figure 41. The third and fourth circles are drawn to the same radius from the intersections of the first and second circles, figure 42.

Figure 43 shows the four circle construction separately and enlarged for clarity. Two lines, shown in magenta, pass through intersections of the circles and extend to mark two points on the circumference of circle 1. Figure 44 shows how the two points are the axes for the fifth and sixth identical radius circles, shown in dashed black line, and these complete the scribed symbol.

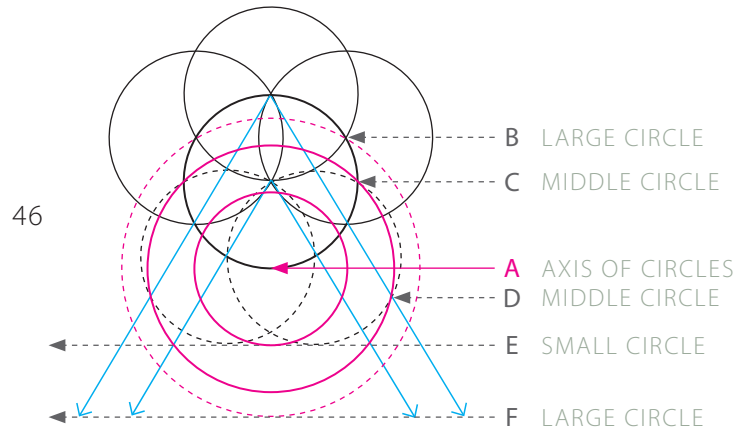
The symbol's development into the doorhead canopy commences with figure 45 which, to make the construction clearer, is drawn on a vertical axis and at a larger scale.

Two new alignments are drawn from the top of the central circle at point A through the circle intersections at B and C and beyond. Two lower parallels are drawn from the axis of the central circle at point D and beyond. The alignments are shown in blue line. The fact that points A and D are the top and centre of the central circle is significant because



the next step in the construction is from the central circle's base. The development is therefore from three points on the central circle's vertical diameter, one at the axis and two on opposite poles of the circumference.

In figure 46, three concentric circles, shown as magenta line, are drawn from an axis at point A at the central circle's lower pole. The largest dashed circle passes through points where the top black circle intersects with the central circle at level B. The large solid line circle passes through the points of intersection where the dashed black circles cut the central circle at level C and where they cut the blue angled lines at level D. The small circle's lower pole kisses the tangent to the lower poles of the dashed black circles at level E. A tangent to the lower pole of the large dashed circle, at Point F, terminates the blue lines.



With the construction lines complete it is possible to select those that give the door-head canopy. In figure 47, bold magenta highlights the essential lines while the remainder of the construction lines are shown as half tone. The bold magenta lines define the inner and outer boundaries of the Tre-sham's unusually shaped door-head canopy.

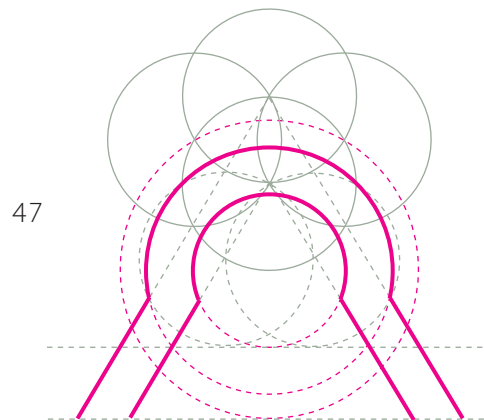
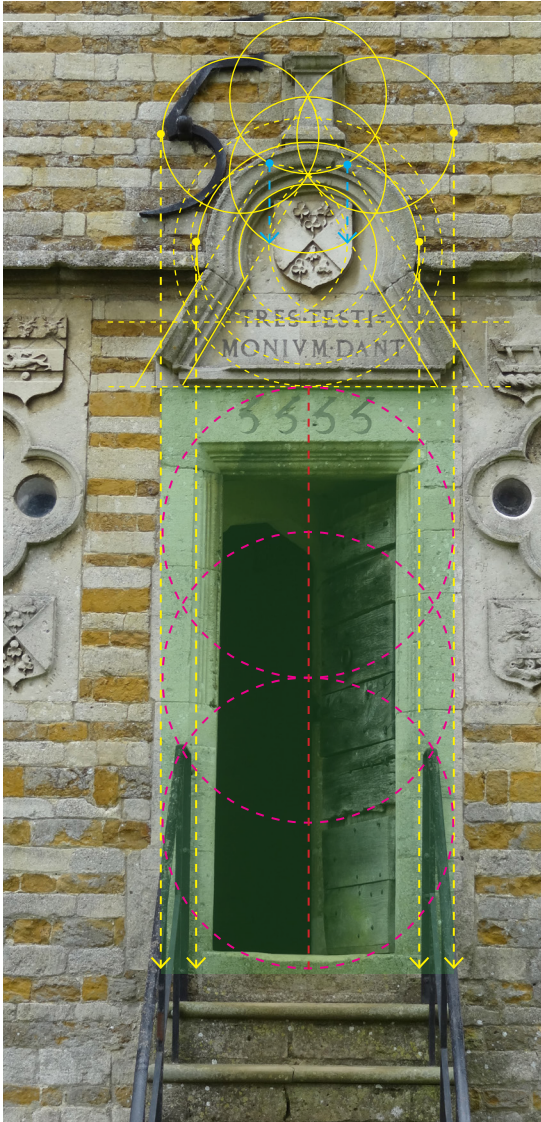


Figure 48 shows the door-head geometry superimposed over a photograph of the actual structure. Plumb lines from the geometry define the boundaries of the door's jambs.



Tresham's sophisticated geometrical journey from the stone beneath the aumbry to the final form of the door-head canopy follows an inescapable theological logic based on the Trinity. There are three stages ~

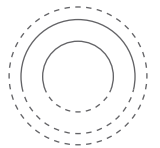
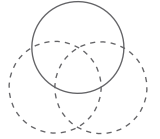
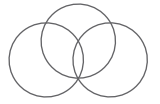
In the first stage the stone's perimeter is defined by a triple daisy wheel proportion, secondly the area of the stone is divided using diagonal geometry to give nine sectors (or triple Trinity) and, finally, the geometrical symbol is scribed, see figures 38, 39 and 44.

The scribed geometry generates nine circles. The first stage of six interlaced circles can be viewed as two trefoils while the final group are concentric, see drawings in margin.

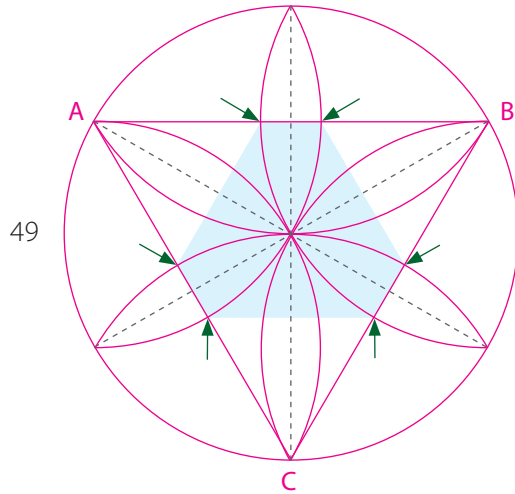
The process also moves through three stages, commencing with stone cut to its preliminary rectangular shape, evolving through the development of the geometry and then cutting full scale masonry to its final, architectural shape.

The full door case, shown in green tone in figure 48, is a double square, a proportion defined by tangents to a three circle sequence.

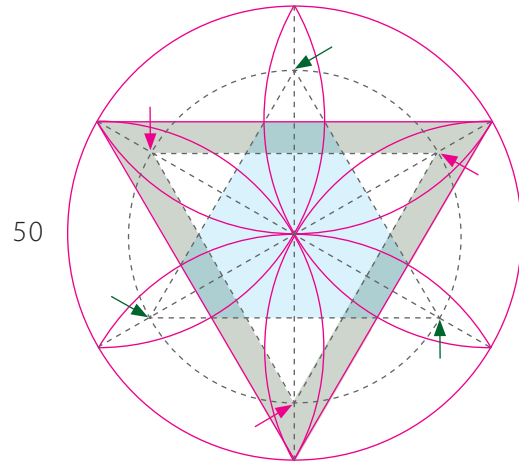
The geometrical development of the door-head, shown here a step at a time for clarity, would in reality have been a single drawing, almost certainly scaled up from the stone beneath the aumbry to full size for the masonry of the building itself, using dividers. The symbol was clearly intended, like a modern time capsule, to carry Tresham's geometrical message forward into an unforeseeable future.



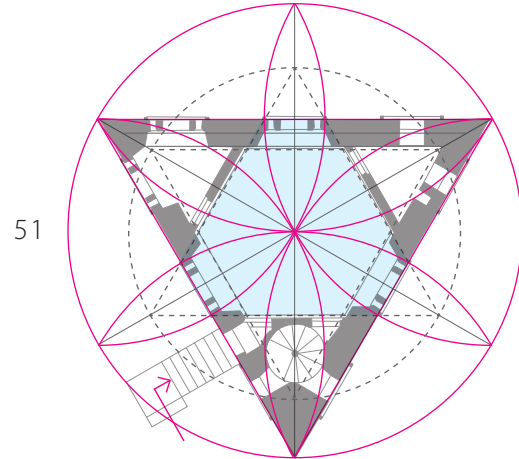
Tresham's daisy wheel floor plan



The floor plan, like the upper floor fenestration, is developed from daisy wheel geometry. In figure 49 a triangle, ABC, is drawn between alternate petal tips on the wheel's circumference. The triangle ABC cuts the remaining petals at the points indicated by six green arrows and the points are linked to form a truncated hexagon, shown in blue tone.

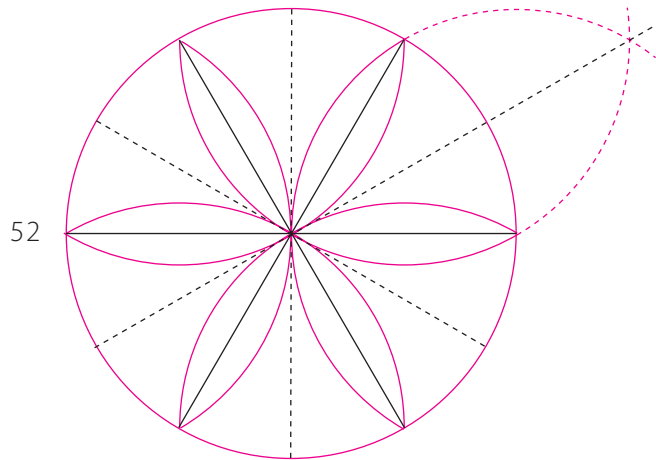


In figure 50 the truncated hexagon is extended as far as the centre lines of the petals to form a triangle, indicated by three green arrows. A circle drawn through the triangle's angles cuts the centre lines of the petals at the three alternate points shown by the magenta arrows. A triangle drawn between the magenta points runs parallel to the original triangle ABC and the space between them



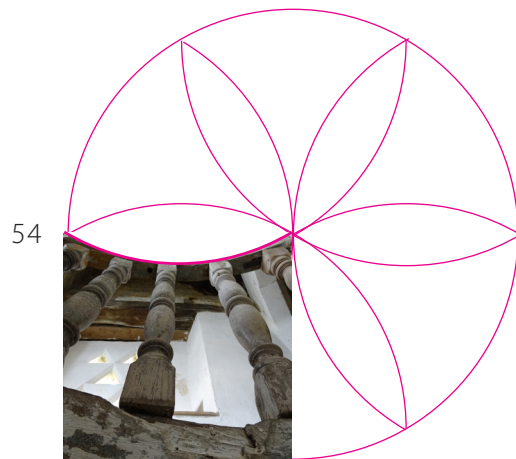
determines the wall thickness, shown in grey tone. Figure 51 shows the geometrical construction superimposed over the floor plan.

The spiral stair and stairhead bannisters

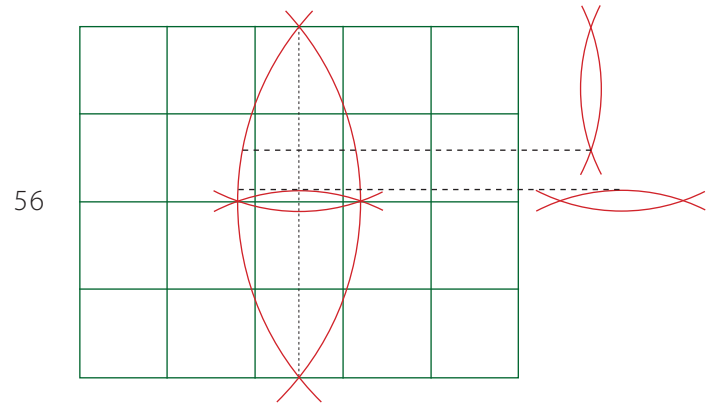
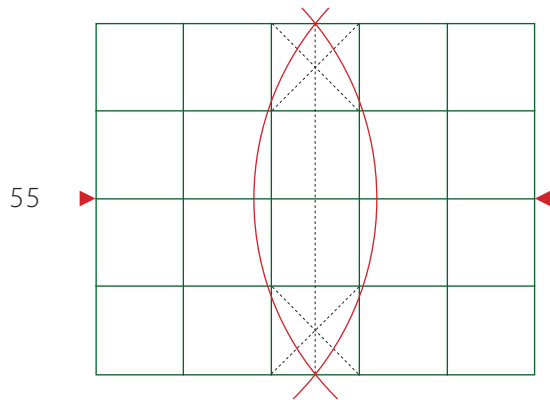


The spiral masonry stair that links the three floors is also designed using daisy wheel geometry. Drawing the wheel by compass automatically divides the wheel's circumference into six equal 60° sectors and these, halved to 30° (by arcs drawn from adjacent petal tips), give twelve treads around the stair's axis, figure 52.

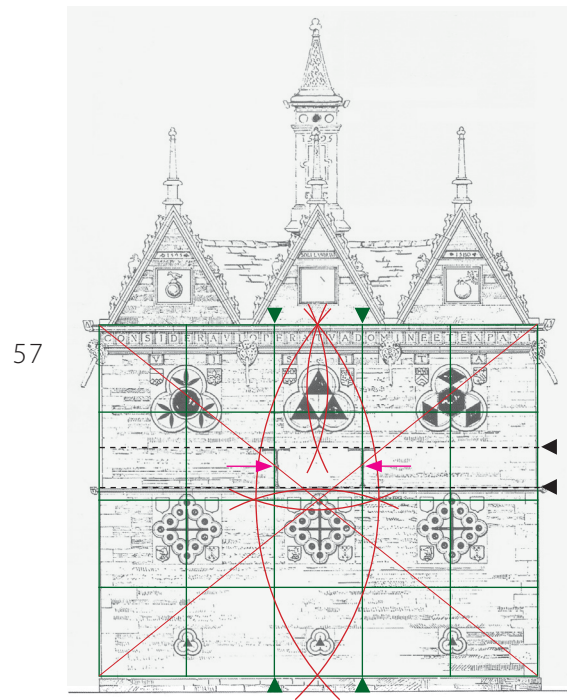
As the stairs emerge onto the top floor from the masonry cylinder surrounding them, the open floor edge is protected by a balustrade of lathe-turned oak posts, polished to a glossy patina by the hands of visitors over the centuries, figure 53. Looking up from a lower point on the stair's central axis, it can be seen that the balustrade handrail follows the curvature of a daisy wheel petal from the wheel's axis to its circumference, figure 54.



Thomas Tresham's TT monogram



Thomas Tresham's TT monogram has a geometrically defined location on the building's facade. Figure 55 shows how a vesica piscis is drawn from opposite ends of the 4 x 5 grid centre line to fill the grid's full height and figure 56 shows a smaller vesica drawn from the large vesica's intersections to fill its horizontal width. The width is also marked vertically from the large vesica's top intersection. The two small vesicas give the location of two horizontals, indicated by black arrows in figure 57, that define the height of the TT monogram and with the lower also defining the string course. The horizontal strokes of the monogram are bounded by the large vesica and the vertical strokes are defined by the grid lines indicated by green arrows in figure 57. Diagonals drawn across the grid cut the vertical strokes of the TT monogram at their centre points, indicated by the magenta arrows and marked on the building by small swellings in the metalwork.



FOOTNOTES

1

There are two excellent small publications on Sir Thomas Tresham and Triangular Lodge -

Rushton Triangular Lodge

Department of the Environment official guidebook
By Sir Gyles Isham, FSA

HMSO, London 1970 SBN 11 670061 0

Rushton Triangular Lodge

By Mark Girouard

English Heritage 2004 ISBN 1 85074 819 5

Telephone 01536 710761

2 and 3

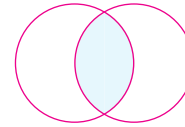
The vesica piscis, shown above photographs 2 and 3, is formed by the overlap of two equal radius circles that pass through each other's centres. The Latin name, meaning *bladder of a fish*, derives from its fish-like shape. It is also known as a *mandorla* from the Italian for the similarly shaped almond.

The photographs, taken at Ely Cathedral, show vesicas above the nave's high western arch, figure 2, and in the tympanum of the Prior's Door, figure 3. Tresham was imprisoned for a time at Ely and there is a possibility that either he knew of these vesicas or, depending on the conditions of his imprisonment, may have witnessed them personally. What is certain is that, as a Christian, he would have understood their symbolic meaning.

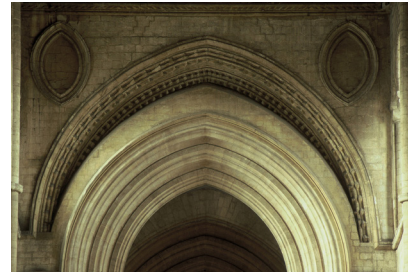
4

Seen in shadow against the incoming window light in one of the small triangular closets, the geometrical symbol is almost invisible to the naked eye. However, raking light from a hand torch held close to the wall plane illuminates the surface of the stone and reveals the scribed circles of the symbol as shadow.

FIGURES



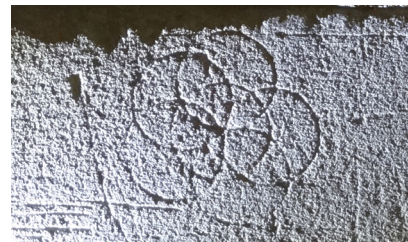
2



3



4



www.historicbuildinggeometry.uk

